

**A Thesis Submitted for the Degree of PhD at the University of Warwick**

**Permanent WRAP URL:**

<http://wrap.warwick.ac.uk/91957>

**Copyright and reuse:**

This thesis is made available online and is protected by original copyright.

Please scroll down to view the document itself.

Please refer to the repository record for this item for information to help you to cite it.

Our policy information is available from the repository home page.

For more information, please contact the WRAP Team at: [wrap@warwick.ac.uk](mailto:wrap@warwick.ac.uk)



# Essays in Political Economics

by

**Raghul Srinivasan Venkatesh**

**Thesis**

Submitted to the University of Warwick

for the degree of

**Doctor of Philosophy**

**Department of Economics**

September 2016

THE UNIVERSITY OF  
**WARWICK**

# Contents

<b>Acknowledgments</b>	<b>v</b>
<b>Declarations</b>	<b>vi</b>
<b>Abstract</b>	<b>vii</b>
<b>Abbreviations</b>	<b>viii</b>
<b>Chapter 1 A Theory of Activism and Polarization</b>	<b>1</b>
1.1 Introduction . . . . .	1
1.2 Related Literature . . . . .	7
1.3 Model . . . . .	11
1.3.1 <i>Median Voter Subgame</i> . . . . .	15
1.3.2 <i>Activist Contribution Subgame</i> . . . . .	16
1.3.3 <i>Symmetric platforms and supply of activism</i> . . . . .	19
1.3.4 <i>Candidate platforms</i> . . . . .	20
1.4 Comparative statics . . . . .	22
1.4.1 Partisan gap and candidate polarization . . . . .	23
1.4.2 Other results . . . . .	27
1.5 Welfare analysis . . . . .	29
1.5.1 Rationale for becoming an Activist . . . . .	32
1.5.2 Strength of party organization . . . . .	35

1.6	Extensions . . . . .	36
1.6.1	Activism in Noisy campaigns . . . . .	36
1.6.2	The role of soft money . . . . .	37
1.7	Conclusion . . . . .	39
<b>Chapter 2</b>	<b>Cheap Talk and Strategic Substitutability</b>	<b>41</b>
2.1	Introduction . . . . .	41
2.2	Leading Example . . . . .	45
2.3	The Model . . . . .	47
2.3.1	Full Revelation . . . . .	50
2.4	Partial Revelation . . . . .	53
2.4.1	Multiplicity of threshold equilibria . . . . .	55
2.5	Comparative Statics . . . . .	57
2.6	Welfare . . . . .	58
2.7	Conclusion . . . . .	61
<b>Chapter 3</b>	<b>An Informational Theory of Alliance Formation</b>	<b>63</b>
3.1	Introduction . . . . .	63
3.2	Model . . . . .	65
3.2.1	Communication Round . . . . .	66
3.2.2	Action round . . . . .	67
3.2.3	Equilibrium Definition . . . . .	68
3.3	Equilibrium Characterization . . . . .	69
3.3.1	Characterization of equilibrium contributions . . . . .	70
3.3.2	Full Information aggregation with resource constraints . . . . .	74
3.4	Conclusion . . . . .	76
<b>Appendix A</b>	<b>A Theory of Activism and Polarization</b>	<b>77</b>
A.1	Proofs - Benchmark model . . . . .	77

A.1.1	Proof of Lemma 1 . . . . .	77
A.1.2	Proof of Lemma 2 . . . . .	77
A.1.3	Proof of Lemma 3 . . . . .	78
A.1.4	Proof of Proposition 1 . . . . .	78
A.1.5	Proof of Proposition 2 . . . . .	79
A.1.6	Proof of Proposition 3 . . . . .	80
A.1.7	Proof of Proposition 4 . . . . .	81
A.1.8	Proof of Lemma 4 . . . . .	82
A.1.9	Proof of Lemma 5 . . . . .	82
A.1.10	Proof of Proposition 5 . . . . .	82
A.1.11	Proof of Proposition 6 . . . . .	83
A.1.12	Proof of Proposition 7 . . . . .	83
A.2	Proofs - Extensions . . . . .	84
A.2.1	Activism and Noisy Campaigns . . . . .	84
A.2.2	The role of big money in campaigns . . . . .	85
<b>Appendix B Cheap Talk and Strategic Substitutability</b>		<b>87</b>
B.1	Formal definitions of PBE and PRE . . . . .	87
B.1.1	PBE: . . . . .	87
B.1.2	PRE: . . . . .	88
B.2	Proofs . . . . .	89
B.2.1	Proof of Proposition 9 . . . . .	89
B.2.2	Proof of Proposition 10 . . . . .	90
B.2.3	Proof of Proposition 11 . . . . .	91
B.2.4	Proof of Proposition 12 . . . . .	92
B.2.5	Proof of Proposition 13 . . . . .	95
<b>Appendix C An Informational Theory of Alliance Formation</b>		<b>97</b>
C.1	Full Information Aggregation . . . . .	97

C.1.1	Characterization of second-stage contributions with unrestricted domain . . . . .	98
C.1.2	Proof of Theorem 1 . . . . .	105
	<b>Bibliography</b>	<b>107</b>

# Acknowledgments

At the outset, I would like to thank the numerous staff and faculty members at the Department of Economics for helping me navigate through the various processes and procedures during my time at Warwick.

A special thanks to all the wonderful friends and colleagues at the Economics Department for all the intellectual discussions over the course of my PhD. All the pleasant memories of the times spent with them at the Department, and outside, shall remain with me for life.

I would also like to extend my sincerest thanks to the Theory Faculty for maintaining a highly creative and intellectual atmosphere at the Economics Department. Throughout the last five years, I have had the pleasure of discussing and presenting my research with them, and have greatly benefited from their insights.

I am forever indebted to my thesis supervisor Francesco Squintani for his sagacity, patience, and guidance over the course of the last five years. He has been a source of inspiration and has passed onto me a wealth of knowledge, for which I am deeply grateful. His intellect and profundity has had a positive impact on my research, and has made me a better researcher. Beyond this, he has been a mentor to me in the truest sense of the word, for which I am eternally thankful.

Finally, I would like to thank my family for being highly supportive of my endeavors and for providing me emotional support right through the thesis.

Thanks to Amma and Appa for their unconditional love.

# Declarations

I would like to declare that all the work in this thesis have been done as part of my doctoral thesis at University of Warwick. Further, this work has not been submitted for a degree at another university.

The first two chapters are entirely my own work. The third and last chapter of my thesis is joint work with Francesco Squintani. The basic idea and the theoretical framework came out of joint deliberations, while all the analysis was mostly my own work.



# Abstract

In Chapter 1, I develop a theory of activism and polarization in the context of electoral competition. I establish that the relationship between ideological polarization of activists and political polarization depends critically on the activists' willingness to engage in the campaign. Specifically, when the willingness to engage is within a threshold, increased partisanship among activists reduces political polarization – meaning candidates compromise rather than diverge. Welfare results suggests that partisan gap could hurt voters when activists have a high willingness to engage.

In Chapter 2, I analyze a modified version of the classic Crawford-Sobel (CS) model of strategic communication between an informed Sender and uninformed Receiver, with the following two innovations: both players now take actions, and they are strategic substitutes. Contrary to the CS setup, the modified game does allow for perfect information revelation. When the Sender is able to compensate sufficiently for every state, there is full information revelation. When this is violated, there are only partial revelation equilibria. Under partial revelation, the Sender reveals information upto a threshold state, and pools beyond this threshold, resulting in loss of information. Welfare analysis suggests that a partial revelation equilibrium with higher threshold is both ex-ante Pareto efficient and interim efficient.

In Chapter 3, we develop a model of alliance formation between players with the following features: substitutability in actions; a need for information sharing; preference heterogeneity; and, resource constraints. The main result is the following: with public communication, there is full information aggregation as long as preferences of players are sufficiently cohesive. We derive a precise bound to characterize cohesiveness, and provide an informational rationale for alliance formation.

# Abbreviations

BDS Bernhardt, Duggan, and Squintani (2009)

CS Crawford and Sobel (1982)

HTIS Highest type incentive to separate

IC Incentive compatibility

LTIS Lowest type incentive to separate

MPR McCarty, Poole and Rosenthal (2016)

PBE Perfect Bayesian equilibrium

PRE Partial Revelation equilibrium

WTE Willingness to engage

# Chapter 1

## A Theory of Activism and Polarization

### 1.1 Introduction

*"People at the top might devote time and resources to supporting a political party strongly opposed to redistribution. People at the bottom would have an opposite response."* - McCarty, Poole and Rosenthal, *Polarized America*

Electoral competition between political candidates is a vital component of a thriving representative democracy. Candidates compete by committing to a choice of platforms with an aim to capture power, and implement them once in office. When candidates offer highly divergent platforms, this causes political polarization that may be harmful to the electorate as a whole<sup>1</sup>. A key role of citizens in a democratic society, then, is to effectively participate in the political process in order to promote consensus and compromise, and keep polarization in check.

Traditionally, citizens in a democratic society participate either through the

---

<sup>1</sup>Some harmful consequences of greater polarization are legislative gridlocks on public policy issues, socio-economic problems, among others. See Chapter 6 in McCarty, Poole and Rosenthal [50] (henceforth MPR).

act of voting (passive participation), or by becoming activists (active participation) and associating themselves with political parties. Since the latter form of participation is useful for candidates as a tool of mobilization and influence, party activists, through their costly participation<sup>2</sup>, have the ability to affect the platform choice of candidates. Party activists, on the other hand, may themselves be driven by an ideological inclination. This partisanship between activists, combined with the candidates' dependence on them, could then increase the levels of political polarization, to the detriment of societal welfare.

Take, e.g., the Pew Research Center's study<sup>3</sup> in 2014 that documents this partisanship, and I quote - "Today, 92% of Republicans are to the right of the median Democrat, and 94% of Democrats are to the left of the median Republican.". Further, it adds, "But on every measure of engagement, political participation is strongly related to ideology and partisan antipathy; those who hold consistently liberal or conservative views, and who hold strongly negative views of the other political party, are far more likely to participate in the political process than the rest of the nation. This results in a consistent "U-shaped" pattern, with higher levels of engagement on the right and left of the ideological spectrum, and lower levels in the center. ".

This kind of partisanship suggests that activists may demand policies to compensate for their costly effort and candidates may find it beneficial to cater to their preferences. This way, party activists may themselves become a potential source of polarization. Aldrich [1] [2], in his seminal work, develops a rational-choice theory that predicts precisely this kind of partisan identification to emerge among party activists. His work also analyzes conditions under which this partisan

---

<sup>2</sup>The critical role played by party activists have been well documented in the field of political science. For more on the role of political parties and party activists, read Aldrich [3], Hershey [63], Norris [58].

<sup>3</sup>See <http://www.people-press.org/2014/06/12/political-polarization-in-the-american-public/>. Also, for a more recent study on partisanship, see <http://www.pewresearch.org/fact-tank/2016/01/27/the-demographic-trends-shaping-american-politics-in-2016-and-beyond/>.

gap may diverge. Though Aldrich’s work provides a framework for studying the motivations for becoming an activist, questions of how this partisanship influences political platforms, what affects the extent of participation by partisan activists, and the welfare effects of activism remain unanswered, and pertinent, especially so in the current context of the polarization debate surrounding American politics.

I develop a theory of activism and political participation that captures these intuitive trade-offs by integrating party partisanship, the role of activists, and candidates’ platform selection in a model of electoral competition. My theory incorporates three key features – i) candidates care about ideology and benefits of office (they are “responsible”, in the spirit of Calvert [22] and Wittman [72]); ii) activists are ideological price-takers (they do not influence platforms directly, as modeled by Aldrich); iii) activists influence voters but face participation costs.

The political process proceeds as follows: candidates simultaneously announce platforms, party activists expend effort to influence voters given the set of platforms, and (median) voter decides whom to vote for; in that order. Candidates, when announcing platforms, and activists, when deciding on levels of participation, are unaware of the voter’s preferred policy, which is determined from an uniform distribution. Activist participation plays a role of influence, in the sense that their effort directly affects median voter’s utility, by shifting their preferences towards a candidate. Two important trade-offs emerge in this setup. Candidates trade-off their ideology to elicit greater participation from activists. And, activists trade-off benefits of participation (improving the winnability of their party’s candidate) and the costs of doing so. Together, these twin trade-offs provides for a novel set of results.

First, I find that political polarization decreases in the presence of party activists, compared to the case in which there were no parties. This suggests that parties (and activists) as an institution have a moderating influence on platforms

and thereby help bringing about compromise between candidates<sup>4</sup>. Second, my finding suggests that increased partisanship among party activists does not necessarily increase platform polarization. Specifically, when activists' *willingness to engage* in the political process is above a threshold, an increase in partisanship leads to greater convergence in candidate platforms, resulting in lesser polarization. Further, when the willingness to engage falls below the threshold, greater party polarization leads to greater political polarization. Rather surprisingly, at precisely this threshold, platform polarization is independent of the extent of partisanship among parties.

An important implication of this result is that a widening partisan gap is neither necessary nor sufficient for causing increased polarization in platforms. In fact, my analysis suggests that a combination of partisanship and decreased willingness to engage<sup>5</sup> is what drives polarization. The fact that activists are ideologically risk-averse and prefer moderation implies that as long as they have a greater sense of engagement in the campaign process, more moderate platforms would prevail as an equilibrium of electoral competition.

I derive a rich set of comparative statics results with respect to the exogenous parameters of the model. Specifically, I find that an increase in the relative demand for activism, or a decrease in the uncertainty regarding median voter's ideal point both lead to more moderation in equilibrium. The reason for this is twofold. Any increase in the demand parameter or reduced uncertainty about the median voter increases the relative importance of activism in the elections. This, combined with the fact that party activists have a moderating influence on platforms, implies that

---

<sup>4</sup>This "*compromise effect*" of party activism is along the lines of what Aldrich predicted in his work.

<sup>5</sup>In order to motivate the "*willingness to engage*", consider the following statement published in the Time magazine (September, 2010) that contrasts Obama's presidential campaign of 2008 and 2012: "The outfit that put upwards of 8 million volunteers on the street in 2008 — known as Organizing for America — is a ghost of its former self. Its staff has shrunk from 6,000 to 300, and its donors are depressed: receipts are a fraction of what they were in 2008. Virtually no one in politics believes it will turn many contests this fall." It suggests a marked difference in the enthusiasm levels of supporters during the two campaigns, and provides a glimpse of the type of engagement that is associated with activists.

candidates would compromise in equilibrium.

Next, I investigate the welfare properties of polarization on voters and activists. I find that while voters prefer a moderate level of polarization, the activists' welfare is maximized when there is Downsian convergence in platforms. Though the former result is a direct implication of the analyses of Bernhardt et al. [19], the addition of activists and their preference for perfect convergence creates interesting trade-offs. I find that the addition of activists lowers the socially optimal level of polarization compared to the case when there were no activists. This way, the overall welfare maximizing level of polarization is smaller in the presence of costly participation.

Further, increased partisanship among party activists may improve overall welfare. This stems from the fact that when activists' willingness to engage is below a threshold, increased partisanship also increases polarization. But, when the levels of polarization is below the social optimum, the overall welfare is increasing. Both these factors guarantee that when the level of polarization is very low, increasingly partisan parties with low willingness to engage could actually result in providing more choice to the voters and thereby improve welfare. This result suggests that democratic societies with greater (lesser) barriers to political participation could actually benefit from increased partisanship, as long as the existing choices provided by candidates were highly similar (dissimilar).

The study of welfare effects suggests that voters and activists are welfare-indifferent at an unique *no-arbitrage* equilibrium. This result implies that any level of polarization that is smaller (larger) than the no-arbitrage equilibrium provides the activists (voters) with a welfare surplus. I intuitively characterize this surplus and find that whereas the voters' welfare surplus is increasing in the level of polarization, the activists' is decreasing in the same. The presence of a welfare surplus has an implication that this may be interpreted as a rationale for becoming an activist. Besides, any positive surplus for party activists is a measure of how strong parties

are relative to the voting electorate in a society. Therefore, for any equilibrium level of polarization, the activists' welfare surplus provides a measure of the strength of party organizations.

Finally, I explore the possibility of reconciling the social optimum and the no-arbitrage equilibrium. I show that there exists a level of benefit of party association that ensures that the socially optimal policy coincides with the no-arbitrage equilibrium. This parameter is increasing in the level of partisanship and the relative demand for activism. I find that when the benefits of party association falls below this threshold, implementing the social-optimum policy provides voters with a welfare surplus compared to the activists.

My analysis suggests a possible institutional reform that could potentially address the issue of party strength in democracies. Several studies (e.g., Norris [57] [58], Dalton and Wattenberg [27]) have pointed out the phenomenon of decreasing political engagement and weakening party structures in advanced economies. My analysis suggests that this may be a consequence of the lack of incentives (welfare-surplus) from associating with political outfits. This may lead to weakening of party organization to the point that participating is no more optimal from the standpoint of welfare. A useful intervention would be to design political institutions that would incentivize active participation (strengthening party organizations) and at the same time create the right level of demand for such participation. This strengthening of parties could shift the welfare gains more towards activists, and away from voters.

In Section 5, I consider two extensions to the baseline model. In the first, I investigate the role of activism in a noisy campaign, in the sense of Austen-Smith [10]. Activism, instead of influencing voters' utility, instead plays an informative role. The median voter observes an imperfect (noisy) signal of the actual platform, and greater levels of activism reduce the variance of this noise, rendering platforms more informative. The willingness to engage in this case is dependent on the efficiency of activism and the participation aversion of activists. The results yielded by



this modified setup is similar to the original game. Specifically, the noisy campaign game yields an unique equilibrium and all the comparative statics results go through unchanged.

In the second extension, I study the role of soft money in the electoral competition game. Apart from seeking the support of party activists, candidates are also endowed with a fixed level of campaign money and they use this for influencing voter preferences. In this sense, money and activist participation both perform the same role and supplement each other. That is, money and activist effort are both substitutable goods. In this setting, my analysis suggests a novel crowding-out effect of soft money on activism – greater the available pool of soft money, lesser the participation of activists, and hence, more polarized the platforms. My analysis suggests that introducing public funding of elections could be useful as a potential policy intervention. Public funding restricts the resources available, and thereby limits the crowding-out effect of big money. My model suggests that this would restore the dependence between candidates and activists, and bring out greater moderation in platforms as a result.

The rest of the paper is organized as follows. In Section 1.2, I briefly discuss related literature. Section 1.3 presents the benchmark model and characterizes the equilibrium of the electoral game. Section 1.4 discusses the main results from comparative statics analysis. Section 1.5 details the welfare results and Section 1.6 contains two extensions to the model. A brief discussion and concluding remarks follow in Section 1.7. All proofs are confined to the appendix.

## 1.2 Related Literature

My paper, at its core, is a model of electoral competition which induces platform separation in equilibrium. Several models have explored the idea of platform diver-

gence<sup>6</sup>. This paper looks into electoral activism as a possible channel for divergence, and more importantly, the main goal of my work is to theoretically investigate the impact of partisanship on polarization.

The work of Aldrich [1] is a natural starting point for my analysis. Aldrich derives two important results with respect to the existence of "party cleavages": firstly, within a party, the distribution of activists is cohesive in ideology; and secondly, across party lines, ideologies are distinctly polarized in terms of their distributions. In this paper, I use the modeling assumption about the existence of partisan cleavages<sup>7</sup> to analyze the extent of participation and its impact on political competition. I specifically concentrate my analysis on the relationship between activist polarization as envisaged by Aldrich, and political polarization.

My model is built on the work by Bernhardt, Duggan and Squintani [19] (henceforth BDS). They consider the case for responsible parties in the presence of uncertainty around the median voter's ideal policy. They present an important normative result – a small level of polarization actually improves voter welfare. I introduce political participation (through activists) to this setup and consider the impact of activist polarization and participation on candidate platforms. Contrary to their finding, in the presence of activists, the voters' welfare maximizing level of polarization is never the social optimum. This is because activists in my model always prefer greater convergence and therefore, overall welfare is higher at lower levels of polarization, as compared to BDS.

This paper is closely related to the work on political participation of voters. On the theoretical side, this strand of literature could be broadly categorized into two classes of turnout models<sup>8</sup> – turnout driven by costly voting (Riker and Ordeshook [65], Palfrey and Rosenthal [61] [62], Morton [55], and Feddersen and Sandroni [32]

---

<sup>6</sup>See Bernhardt et al. [18], Osborne and Slivinski [59], Besley and Coate [21], Gul and Pesendorfer [38], Ashworth and Mesquita [8], Aragones and Palfrey [7]

<sup>7</sup>The preferences of candidates in my model are derived from Bernhardt et al. [19], who make a case for responsible parties. See Calvert [22], and Wittman [72] for candidate motivations.

<sup>8</sup>See Herrera, Morelli and Nunnari [42] and Herrera, Morelli and Palfrey [43] for work that relate turnout to institutional arrangements.

[31]) and turnout driven by candidates (Shachar and Nalebuff [66], Herrera and Martinelli [41], Herrera et al. [40], Feddersen and Gul [30]).

The classic work of Riker and Ordeshook [65] introduces the idea of a "calculus of participation" based on costs and benefits of voting; Palfrey and Rosenthal [61] consider a strategic voter model in a game theoretic setup; in Feddersen and Sandroni [32] [31], the voter is driven to vote by a sense of civic duty (positive payoff from voting), or an ethical cost from abstaining; and Morton considers group decisions based on cost and benefits of voting. In these models, the act of voting is a costly decision, and voters makes a decision to participate based on this. In my model, the act of voting is costless (passive participation), but activism is costly. Party activists undertake costly campaign effort just so their preferred candidate wins. This shift the costs from the voter to the activist and hence, fundamentally changes the nature of trade-offs. In particular, candidates announce policy platforms taking into account both the voter's and activists' preferences.

In the latter set of models, the act of voting is not costly, but getting the vote out is. Candidates therefore expend costly effort (spending) in order to ensure voter participation. The model close to my setup is the one by Herrera and Martinelli [41] and Herrera et al. [40]. In the first, citizens decide whether to participate by becoming an activist (influencer), who can then mobilize support among the rest; in the second, candidates announce platforms and spend costly effort to increase turnout of voters. The model of activism differs from this in two ways. Firstly, my paper is not a model of turnout but one of costly influence. Secondly, the effort is borne not by candidates but by activists who belong to their party. This is a natural assumption to make. Party organizations do play an important role in voter mobilization<sup>9</sup>. However, my model assumes a role of influence for activists. Recently, on the empirical front, Madestam et al [49] investigated whether political protests

---

<sup>9</sup>On the empirical front, Gerber and Green [34] show that political activism (as measured by canvassing, phone calls etc.) increases voter turnout in elections. McClurg and Holbrook [44] investigate the role of campaign mobilization on driving core partisan groups to participate in voting.

(or activism) alter voter preferences and impact political outcomes in the context of Tea-party activism in US during the 2010 midterm elections. They find strong evidence<sup>10</sup> for a persuasive role of activism. Specifically, the precise mechanism they identify is one in which activism influenced voters' ideological views, leading to an increase in the percentage of votes for the candidates supported by the activists. This influence role that I incorporate in my model is one of the important modeling contributions of my work, compared to the existing work in the literature (described above).

My model is also closely related to the work on direct informative role<sup>11</sup> of campaign spending, notably, by Austen-Smith [10]. In Austen-Smith [10], candidates simultaneously announce policy, and elicit contributions from two firms. The electoral game analyzed by Austen-Smith differs from mine in two aspects. They consider an informative role whereas I focus on the persuasive role of activism<sup>12</sup> (in that they affect median voter's ideological preferences). This apart, the contribution decision of firms in Austen-Smith are not constrained in that there is no party affiliation and donors can choose to contribute to either candidate. The motivations for activists in my work is to associate with a single party (left or right) and strictly support only their party's candidate.

Finally, it is important to differentiate activism from interest groups or lobbies. The literature on campaign contributions and influence seeking by interest groups or lobbies is vast and has been extensively studied. Baron ([16],[15]), Grossman and Helpman ([?], [37],[36]), Bernheim and Whinston [20], Austen-Smith [13] investigate various aspects of influence seeking by interest groups. The distinctions

---

<sup>10</sup>Their main finding is that political activism had significant multiplier effects in terms of affecting the number of votes secured by Republican candidates, and also resulted in more conservative stances by policymakers in congress. They conclude "...these results are consistent with larger political protests creating a stronger political movement that is able to more effectively persuade the populace about its policy agenda come election time, which ultimately affects both incumbent behavior and election outcomes".

<sup>11</sup>Coate [24] presents an alternate model of informative campaign spending.

<sup>12</sup>We extend our model to include noisy campaigns, and show that the fundamental predictions on equilibrium polarization and activist participation holds. See Section 6.

between these models and mine is twofold. Firstly, activists are quite different in their objectives, in the sense that their support is ideologically partisan and is not driven by any form of non-partisan or economic considerations. Further, the commitment of activists towards a candidate is temporary and short-term, unlike lobbies or interest groups that have interests once a candidate gets elected. Interest groups and lobbies seek influence of incumbents to get favorable legislations or policies passed, and in this sense, their relationship is not short term. The focus of my work, alternatively, is purely on electoral campaigns and the direct role of activists in influencing political platforms of candidates.

### 1.3 Model

Two candidates, who care about ideology and benefits of office, contest elections on an uni-dimensional policy space  $[-1, 1]$ . Candidate  $L$  has an ideal point  $p_L^C = -\alpha$  and Candidate  $R$  has an ideal point  $p_R^C = \alpha$ , where  $\alpha \in (0, 1)$ . The candidates simultaneously announce policy  $X_i$  (where  $i \in \{L, R\}$ ), and the winning candidate enjoys benefits from office,  $b > 0$ . The winner implements the ex-ante chosen policy. The (symmetric) candidate utilities are given by,

$$U_i^C = \begin{cases} -(X_i - p_i^C)^2 + b & \text{if } i \text{ wins} \\ -(X_{-i} - p_i^C)^2 & \text{otherwise} \end{cases}$$

Candidates, after announcing platforms, seek the support of party activists belonging to each party. The activists' payoff consists of three components. They derive a benefit of party association  $K$ , which is independent of the electoral process and is constant. This benefit  $K$  could be interpreted as a form of patronage, or preferments, or purely social incentives that provides with a positive utility from association. Besides, the two activists  $A_L$  and  $A_R$  have an ideological preference that is different from that of their respective candidate, and given by  $p_L^A = -\beta$  and  $p_R^A = \beta$  respectively. Activists therefore contribute to the electoral process by

supporting their candidate through a costly effort decision. This costly political participation of the party activist is captured a convex cost of participation,  $M(c_i)$ , such that  $M' > 0, M'' > 0, M(0) \geq 0, M'(0) > 0$ . Let  $\gamma_m(c_i) = c_i \cdot \frac{M''}{M'}$  be defined as a measure of relative participation aversion of activists. This provides a measure analogous to risk aversion, except that participation aversion measures the elasticity of the costs involved with participating in the campaign process.

Note that party polarization is defined by the ideological distance between activists<sup>13</sup>,  $2\beta$ . An increase in  $\beta$  could be interpreted as a reflection of more partisan parties (or party activists). For example, greater  $\beta$  could be thought of as more extreme views (on the right and left) on tax policy, gay rights, regulations, or minimum wages, and so on. Given  $\beta$ , the mobilization  $c_i$  is very loosely defined to capture any form of contribution by activists. This could be either direct small donations to candidates or indirect ones, namely, door-to-door canvassing, attending campaign events (national conventions, among others), talking to potential supporters in local districts, and so on. Broadly, any measure of time, effort, or money spent on endorsing and campaigning for the candidate could be accounted for by the variable  $c_i$ . The utility for an activist is therefore given by,

$$U_i^A = \begin{cases} K - (X_L - p_i^A)^2 - M(c_i) & \text{if } L \text{ wins} \\ K - (X_R - p_i^A)^2 - M(c_i) & \text{if } R \text{ wins} \end{cases}$$

Finally, there is a median voter, whose ideal point  $\mu$  is unknown to candidates and activists when they choose platforms and level of participation, respectively. The effort spent by the party activist affects the median voter's utility directly by providing a positive utility according to an "*influence function*"  $P(c_i)$ , where  $P(\cdot)$  is twice continuously differentiable and concave in  $c_i$  such that  $P' > 0, P'' \leq 0, P''' \geq 0, P(0) = 0, P'(0) > 0$ . In a similar vein as before, it is useful to define  $\gamma_p(c_i) = -c_i \cdot \frac{P''}{P'}$  as the relative influence aversion.  $\gamma_p(c_i)$  describes the curvature of the influence function and measures the effectiveness of activist contributions in

---

<sup>13</sup>Aldrich refers to this partisan identification as "*party cleavages*".

influencing median voter's preferences. Finally, I assume that there is uncertainty about the voter's ideal point  $\mu$ , and further that  $\mu$  is distributed uniformly on  $[-\sigma, \sigma]$ .

$$U_m = \begin{cases} -(X_L - \mu)^2 + \eta \cdot P(c_L) & \text{if } L \text{ wins} \\ -(X_R - \mu)^2 + \eta \cdot P(c_R) & \text{if } R \text{ wins} \end{cases}$$

This is a reduced form utility function<sup>14</sup>, to capture the fact that increased participation by activists persuades the voter by shifting preferences towards their favored candidate. Notice that activists have an incentive to contribute since increased mobilization directly affects the utility of the median voter, and thereby, the winnability of their respective candidates. This is the persuasive role of activism that I use in the benchmark case. Contributions reflect the extent of mobilization by activists during the election cycle. The salience of activism is captured by the  $\eta > 0$  parameter. A greater  $\eta$  implies that activist participation is weighed more significantly by the median voter, thereby increasing their relevance in the campaign process. Therefore,  $\eta$  represents the "*demand for activism*" by the candidates during electoral competition. When  $\eta = 0$ , the median voter is unaffected by activism, and the game resembles a variant of the BDS paper in which candidates with mixed motivations compete for an electoral office, in the presence of uncertainty about median voter's ideal preference.

The structure of payoffs illustrates three key points relevant for my analysis. First, candidates care about activists because of their perceived influence on median voter's preferences. The influence function of activism,  $P(\cdot)$ , could be interpreted along the lines of Madestam et al [49]. The effort of activists in mobilizing support for their preferred candidate could be seen as providing a kind of direct payoff to the median voter. Thus, the function  $P(\cdot)$  captures this direct benefit from campaign activism. Second, although the influence of activists on voter preferences is indeed critical for a candidate's chances of winning the election, this influence comes at a

---

<sup>14</sup>Using such an utility form for the voter provides a tractable equilibrium solution to the electoral framework. In the extension presented in Section 4.2, we change this assumption to include noisy campaigns.

cost for the party activist. Since activists themselves are risk averse (face a quadratic ideological loss), incurring a cost of participation in the campaign process implies they trade-off potential benefits of campaigning –increasing the winnability of their party’s candidate– and the participation costs that a campaign entails.

Such participation costs are a way to quantify the effort decisions of activists who involve or engage themselves in the party organization (by attending national conventions, canvassing, leafleting, among other party activities). Finally, given this nature of dependence on activists, candidates who rely on partisan activism during the electoral campaign choose policies to reflect this trade-off. This means that candidates announce platforms taking into account the possibility of greater participation from activists, and the improved chances of winning from their contributions.

Therefore, in this paper, I concentrate my analysis on a political environment in which activists are rational yet ideologically motivated, activism influences voters, but is costly, and candidates respond to these trade-offs by either polarizing or moderating their platforms so as to maximize their expected payoffs. The timing of the game is summarized as follows:

1. Candidates  $L$  and  $R$  simultaneously announce policy platforms  $X_L, X_R$
2. Activists observe platforms, and simultaneously choose contributions  $c_L$  and  $c_R$
3. Nature draws the median voter’s bliss point  $\mu$  from an uniform distribution  $[-\sigma, \sigma]$
4. The median voter observes policy platforms of candidates, contribution of activists, and decides the winner

Observe that this sequence of play highlights the ”*price taking*” behaviour of party activists, as envisaged by Aldrich. We could think of candidates moving first as the equivalent of party conventions, in which the party nominees (or the winners



of the respective primaries) announce their platforms in front of party donors and activists, who then take this as given and decide on contributions for the campaign<sup>15</sup>. All the exogenous parameters  $(\alpha, \beta, \eta, \sigma, b)$  and the functional forms of  $P(\cdot)$  and  $M(\cdot)$  are common knowledge. The equilibrium concept is sub-game perfect Nash equilibria (SPNE) in symmetric pure strategies.

### 1.3.1 *Median Voter Subgame*

The (median) voter chooses the party which gives a higher payoff, ie, the voter prefers candidate  $L$  over candidate  $R$  iff,

$$-(X_L - \mu)^2 + \eta P(c_L) \geq -(X_R - \mu)^2 + \eta P(c_R)$$

Therefore the cutoff  $\mu$ , below which the median voter will vote for party  $L$  is,

$$\hat{\mu}(X_R, X_L, c_R, c_L) = \frac{(X_R + X_L)}{2} + \frac{\eta}{2} \frac{P(c_L) - P(c_R)}{(X_R - X_L)}$$

Let  $\lambda(X_R, X_L, c_R, c_L)$  denote the probability with which candidate  $L$  wins when  $X_L \neq X_R$ . Given the distribution of  $\mu$ , the probability of candidate  $L$  winning is, therefore,

$$\lambda(X_R, X_L, c_R, c_L) = \frac{1}{2} + \frac{(X_R + X_L)}{4\sigma} + \frac{\eta}{4\sigma} \frac{P(c_L) - P(c_R)}{(X_R - X_L)} \quad (1.3.1)$$

Notice that the win-probability of candidate  $L$  is increasing in the contributions from activist  $A_L$ , and decreasing in the contribution of activists  $A_R$ . Fixing one of the side's contribution constant (say  $c_L$ ), and increasing the other ( $c_R$ ) reduces candidate  $L$ 's winnability, and vice-versa ( $\frac{\partial \lambda}{\partial c_R} < 0$ ,  $\frac{\partial \lambda}{\partial c_L} > 0$ ). This provides

---

<sup>15</sup>Notice that elections involve many other facets which have been ignored in this set-up. Instead, by isolating and concentrating on the channel of electoral activism, I intend to draw critical insights on the effects of activism on political polarization.

an incentive for activists to mobilize during the campaign. However, activists also face convex effort costs, and the participation decision essentially balances these two opposing forces.

### 1.3.2 *Activist Contribution Subgame*

Consider the contribution decision of the activist. The activist  $A_{L(R)}$  evaluates the winnability of candidate  $L(R)$ , which in turn is dependent, among other variables, on the difference in contributions of the two activists. Before exploring the contribution decision of party activists, it is useful to establish what happens when candidates converge on the same platform in the first stage.

**Lemma 1.** *When platforms are not differentiated ( $X_L = X_R$ ), the activist subgame has an unique equilibrium in which neither activist participates in the campaign,  $c_L = c_R = 0$ .*

*Proof.* See Appendix A.1 □

This result follows from the costly participation decision of activists. When both party candidates converge towards the same platform, the expected ideological loss is constant for the activists. Since activists in the model share purely ideological motives, any positive effort level cannot be optimal given the costs involved, since  $M(c_i) > 0$  for  $c_i > 0$ . In other words, in the absence of any platform polarization, party activists do not stand to gain ideologically (in expected terms) from participation. On the other hand, when platforms are polarized, it introduces incentives for participation.

**Lemma 2.** *When  $X_L \neq X_R \neq 0$ , each activist chooses a level of contribution given by,*

$$\frac{M'(c_L)}{P'(c_L)} = \frac{\eta}{4\sigma} \cdot [2\beta + (X_L + X_R)]$$

and

$$\frac{M'(c_R)}{P'(c_R)} = \frac{\eta}{4\sigma} \cdot [2\beta - (X_L + X_R)]$$

*Proof.* See Appendix A.1 □

The contribution function represents the extent of mobilization by activists, and is dependent on activists' ideology, the platforms announced by the candidates, the demand parameter for activism, and degree of uncertainty in the median voter's ideology. Observe that the contributions are an increasing function of  $2\beta$ , the polarization in party activists' ideologies. This suggests that there is greater level of participation in elections when parties (and their respective activists) are more polarized. This is fairly intuitive to think about. Any electorate that is polarized care more about the issues and face a higher loss when the other side wins, prompting greater participation in the campaign.

To understand the forces more deeply, take the problem of the activists. When activists are more polarized, it becomes more salient for them to support their candidate, since not doing so would result in the opposite party winning and implementing a more extreme policy. Through greater activism and participation in the campaign process, activists could affect their preferred candidate's chances of winning, and as a result, reduce their own (expected) ideological losses. Since activists are purely ideologically motivated and risk averse, greater polarization between the two parties, *ceteris paribus*, implies that the marginal benefits from contribution is higher. This increased marginal benefit implies contributions by activists must increase in order to offset the marginal benefit. However, this increase in contribution is dampened by the presence of participation costs ( $M'' > 0$ ). The optimal contributions are then determined precisely by the expressions above, balancing both the marginal benefit and marginal costs of increased participation.

The other important point to note is the relationship between contributions of activists and candidates' announced platforms,  $(X_L, X_R)$ . There are two effects

at play - i) *preference for moderation effect*, and ii) *counter-mobilization effect*. Specifically, when one of the candidate becomes more extreme, the party activist supporting the candidate reduces contributions to the campaign ( $\frac{\partial c_R}{\partial X_R} < 0$  and  $\frac{\partial c_L}{\partial X_L} > 0$ ). That is, when either  $X_R$  or  $X_L$  becomes more extreme, the sum  $X_R + X_L$  increases and this indirectly affects the contribution decision of the activist supporting  $R$  or  $L$ . This is the preference for moderation effect which implies that ideologically risk averse party activists tend to have a propensity for moderation. As a result, more extreme platforms are disliked by activists, and a willingness to compromise by a candidate (moving closer to the other candidates' platform, say) increases participation from the party activist supporting that candidate.

In addition to this direct effect, there is an additional indirect effect. Specifically, when one candidate becomes more extreme, the activist supporting the other candidate contributes even more ( $\frac{\partial c_R}{\partial X_L} < 0$  and  $\frac{\partial c_L}{\partial X_R} > 0$ ). This stems from the fact that when a candidate polarizes, the expected ideological loss is higher for the other party's activist, and hence, the marginal benefit of contribution is greater. Activists, therefore, not only care about whether their candidate moves closer to the centre, but equally care about whether the other candidate becomes more extreme. In a way, this incentive to counter-mobilize comes from the fact that when one side adopts a more extreme stance, the expected ideological loss increases, thereby prompting greater countervailing (reactionary) mobilization.

Therefore, given two platforms, when one candidate deviates to a more extreme platform, two things change. First, the activist supporting the other candidate chooses greater contributions so as to compensate for the increased marginal benefits. Second, the activist supporting the extreme candidate decreases participation, in order to compensate for the higher expected ideological losses. Moreover, both these effects reinforce each other and favor compromise over polarization. In this sense, the preferences of party activists are such that it favors greater moderation and compromise from candidates during the campaign process.

### 1.3.3 *Symmetric platforms and supply of activism*

Since we restrict attention to symmetric candidate platforms, it is useful to first analyze the form and structure of the supply function of activism under this formulation. When platforms are symmetric, meaning  $X_R = -X_L$ , the contributions by activists are independent of the announced platforms  $(X_L, X_R)$ , since  $X_R + X_L = 0$ . This implies that equilibrium supply of activism is purely a function of the exogenous parameters of the model  $(\beta, \eta, \sigma)$  and is independent of the extent of polarization in platforms. This property is due to the fact that there is no strategic interaction between ideology and the persuasion function  $P(\cdot)$ . This additive separability in the median voter's preferences implies that as long as the two platforms are symmetric on either side of the political spectrum, the optimal contributions of the party activists are unaffected by the extent of platform polarization.

**Lemma 3.** *When candidate platforms are symmetric, ie  $X_R = -X_L$ , supply of activism is given by  $c_L^*(\beta, \eta, \sigma) = c_R^*(\beta, \eta, \sigma) = c^*(\beta, \eta, \sigma)$  that solves  $M'(c) = \frac{\eta\beta}{2\sigma} \cdot P'(c)$ . Moreover, the following holds:  $\frac{\partial c^*}{\partial \beta}, \frac{\partial c^*}{\partial \eta} > 0$ ,  $\frac{\partial c^*}{\partial \sigma} < 0$*

*Proof.* See Appendix A.1 □

Notice that the equilibrium supply of activism has a simple structure. The characterization equates marginal costs and marginal benefits of contribution, resulting in an unique equilibrium of the activist subgame. Since platforms are symmetric, the contributions of both activists are the same in equilibrium. Further, the marginal benefit is weighted by the term  $\frac{\eta\beta}{\sigma}$ . Whenever the demand parameter of activism ( $\eta$ ) is higher, the supply of activism is also greater. Similarly, when there is lesser uncertainty (or variance) regarding the median voter's ideal point, participation increases. These two results stem from the fact that an increase in the demand for activism or reduced variance in median voter's ideal point shifts the marginal benefit curve up thereby increasing the contributions in equilibrium. On the other hand, as explained earlier, as party activists become more divergent

and the partisan gap increases, there is greater participation among the activists. Therefore, any increase in  $\eta$  and  $\beta$ , or a decrease in  $\sigma$ , leads to greater participation in equilibrium. In some sense, this could be interpreted as a "*stakes are higher*" effect. Any change in these parameters either raises or reduces the stakes in the campaign, which in turn leads to greater or lesser participation, respectively.

Although participation in equilibrium is positive when platforms are polarized, it is nevertheless wasteful, in the sense that both the activists' contributions are equal and therefore do not have any relative impact on the winnability of the candidate. However, the reason why they are positive is precisely because if one activist were to reduce the level of contributions, it decreases their candidate's winnability. The other activist, as a consequence, has a greater incentive to contribute, since the marginal benefits of mobilization exceeds the marginal cost of doing so. This interdependence between winnability and activism prompts either party activists to contribute a positive level in the campaign, even though in equilibrium the two contributions cancel off each other resulting in zero net effect of activism.

#### 1.3.4 *Candidate platforms*

Candidates anticipate contributions and the winning probability as a function of their chosen platforms. A (subgame perfect) Nash Equilibrium strategy for a candidate is a policy platform that maximizes their payoff, given the other candidate's platform choice and the subsequent play of the game. I restrict attention to symmetric candidate platforms. Before characterizing equilibrium platforms with party activists, it would be useful to consider the case when there is no demand for activists, meaning  $\eta = 0$ . This describes a political environment devoid of activists, and the equilibrium is determined by candidates with mixed motivations and median voter uncertainty. The following proposition characterizes the equilibrium platforms in the absence of activism.

**Proposition 1.** *The electoral game without activism has a symmetric equilibrium*

$(-\bar{x}, \bar{x})$  such that, if  $\alpha > \frac{b}{4\sigma}$  then  $\bar{x} = \frac{4\sigma\alpha - b}{4(\alpha + \sigma)}$ ; and if  $\alpha \leq \frac{b}{4\sigma}$  then  $\bar{x} = 0$ .

*Proof.* See Appendix A.1 □

An important point to note in the above proposition is that  $\bar{x} < \alpha$ . This implies that candidates never choose their ideal policy and always moderate in equilibrium. As a consequence, introducing ideologically risk averse party activists further changes the incentives for candidates. On top of targeting the median voter's ideal policy, candidate's also have to cater to the preferences of activists. As I had shown earlier, party activists prefer moderation in platforms in the sense that when one candidate chooses a more extreme platform, he is punished through lesser participation by his own activist, and simultaneously, by greater contributions from the other party's activist. This indicates that candidates trade-off ideology in order to elicit greater participation from activists by moderating platforms in equilibrium, irrespective of the extent of partisan gap between party activists.

**Proposition 2.** *The electoral game with activism has an unique symmetric pure strategy equilibrium in candidate platforms  $(-x^*, x^*)$  that solves,*

$$4(\alpha + \sigma)x^2 - [4\alpha(\sigma - \frac{1}{2}D(c^*(\eta, \beta, \sigma), \eta, \beta)) - b]x + \frac{b}{2} \cdot D(c^*(\eta, \beta, \sigma), \eta, \beta) = 0$$

$$\text{where } D(c^*(\eta, \beta, \sigma), \eta, \beta) = \frac{\eta}{\beta} \cdot \frac{c^* \cdot P'(c^*)}{\gamma_m(c^*) + \gamma_p(c^*)}$$

$$\text{such that if } \alpha > \frac{b}{4(\sigma - \frac{1}{2}D(\cdot))} \text{ then } x^* > 0; \text{ if } \alpha \leq \frac{b}{4(\sigma - \frac{1}{2}D(\cdot))}, \text{ then } x^* = 0.$$

Furthermore,  $\bar{x} \geq x^*$ .

*Proof.* See Appendix A.1 □

The equilibrium with activists is more moderate than one in the absence of it. Parties and their activists therefore act as institutions that build consensus and political compromise. This is an interesting finding since it illustrates an important role for parties in democratic polities. In the absence of activists, candidates with mixed motivations would tend to move away from each other and closer to their preferred platform, causing greater polarization. The presence of a party structure

that supports candidates during elections, therefore, also ensures that the candidate they support cater to their preference for moderation and compromise, thereby resulting less divergent platforms. This tendency to moderate is driven by the risk-averseness of party activists and their costly participation decisions. Activists trade-off costly participation and winnability (and hence, lower expected ideological loss) of the candidate, taking as given the two candidate platforms. Therefore, any increase in polarization in this model could be interpreted as coming from candidates trying to push the platform towards their preferred policy.

Given these trade-offs, it is pertinent to study the relationship between party activism and platform polarization. Doing so requires a way in which to describe the incentives for engaging in activism. During campaigns, party activists engage with potential supporters and the wider public in order to mobilize support for their candidate. This means that the willingness to engage of activists plays a crucial role in determining what platforms are chosen in the first place. When activists have a greater willingness to engage, their participation affects the campaign in a more significant way, and candidates would then have incentives to moderate. I will define this willingness to engage as  $WTE = \frac{1}{\gamma_m(c) + \gamma_p(c)}$ . That is, the willingness to engage is simply the inverse of the sum of participation aversion and influence aversion. This gives us an intuitive way to think about the participation decision of activists in the electoral process. Whether this willingness to engage in the political process reduces candidate polarization as activists themselves become more extreme is of considerable interest and will be analyzed in the next section.

## 1.4 Comparative statics

I will present comparative statics results pertaining to the exogenous parameters of the model –  $(\alpha, b, \eta, \beta, \sigma)$ . The comparative statics of interest is the relationship between party activist polarization and candidate polarization. As the partisan



gap widens, it is plausible for candidates to move further away from each other. However, as I argued earlier, this may depend crucially on the activists' willingness to engage. As they become more extreme, given the risk-averseness of their ideological preferences, more moderation and compromise provides activists with lesser expected ideological loss. This simple intuition, coupled with a high willingness to engage is sufficient to guarantee platform compromise rather than polarization. The following sub-section presents this intuitive finding.

#### 1.4.1 Partisan gap and candidate polarization

**Proposition 3.** (i) *As party activists become more extreme, equilibrium platforms are less polarized, ie,  $\frac{dx^*}{d\beta} < 0$  if  $\frac{1-\gamma_p(c^*)}{\gamma_m(c^*)+\gamma_p(c^*)} - c^* \frac{\gamma'_m(c^*)+\gamma'_p(c^*)}{(\gamma_m(c^*)+\gamma_p(c^*))^2} > 1$ .*

(ii) *As activists become more extreme, equilibrium platforms are more polarized, ie,  $\frac{dx^*}{d\beta} > 0$  if  $\frac{1-\gamma_p(c^*)}{\gamma_m(c^*)+\gamma_p(c^*)} - c^* \frac{\gamma'_m(c^*)+\gamma'_p(c^*)}{(\gamma_m(c^*)+\gamma_p(c^*))^2} < 1$ .*

(iii) *The equilibrium polarization is independent of the ideological preferences of the party activist if  $\frac{1-\gamma_p(c^*)}{\gamma_m(c^*)+\gamma_p(c^*)} - c^* \frac{\gamma'_m(c^*)+\gamma'_p(c^*)}{(\gamma_m(c^*)+\gamma_p(c^*))^2} = 1$ .*

*Proof.* See Appendix A.1 □

The above proposition states that there is a non-monotonic relationship between activist polarization and political polarization of candidates, and the nature of this relation is captured by the participation aversion and influence aversion parameters. Specifically, as the partisan gap between parties widen, it need not result in greater candidate polarization in equilibrium as long as the willingness to engage of party activists is sufficiently high. This stems from the fact that as activists become more extreme, they also suffer a greater ideological loss when the other party's candidate wins the election. As a result, irrespective of candidate platforms, when activists become more polarized, the equilibrium contribution goes up ( $\frac{\partial c^*}{\partial \beta} > 0$ ). But, the level of contribution is also dependent on how polarized the platforms announced by the two candidates are. Candidates, on the other hand, understand

these trade-offs. By moderating their platforms, they can extract greater participation from their activist. The downside is that they move further away from their ideal platform. This tension between preference for polarization on one hand and activism driven moderation on the other implies that equilibrium platforms could either diverge further or converge towards each other.

As a consequence, when activists have a greater willingness to engage (meaning their costs are less steep and their influence function is more steep), candidates would find it optimal to compromise rather than polarize. This is so because when one of the candidate polarizes, it pushes the two effects – counter-mobilization and preference-for-moderation – in favor of the other, less polarized, candidate. When the willingness to engage is high, these two effects are more stronger. This implies that the marginal loss from polarizing further is higher than the marginal benefit of doing so, for the polarizing candidate.

The opposite is true when the willingness to engage is low. In this case, candidates find it optimal to polarize since doing so does not change the participation by enough in order to compensate for the increased gains of moving closer to their preferred platform  $(-\alpha, \alpha)$ . This happens a lower willingness to engage by activists implies their marginal costs from activism is high. Thereby, both the counter-mobilization and preference-for-moderation effects are weaker than the previous case. Candidates understand this trade-off while announcing their platforms. They recognize that a lower *WTE* means that activists do not react to polarization as strongly, and this provides them incentives to move closer to their ideal points.

Finally, when the willingness to engage is such that it satisfies the third condition, candidate polarization is unaffected by increasing partisanship among the activists. These results are of fundamental importance in understanding the intricate relationship between party polarization and candidates' platform polarization. What this proposition implies is that a widening partisan gap between parties is neither necessary nor sufficient for increased polarization in campaigns. What is

important is the interaction between partisan gap and the willingness to engage of activists in the electoral process, meaning, a combination of  $(\beta, WTE)$  is critical in determining whether polarization increases. For example, consider the issue of rising income inequality. This issue may push parties towards taxation policies that are more progressive on one side and much less so on the other, thereby increasing partisanship. However, this does not necessarily guarantee that candidates representing either of these parties would further polarize their platforms to reflect this partisanship. In fact, as my analysis suggests, a compromise could be reached in equilibrium if both parties' willingness to engage in the political process is high enough. To see this more clearly, take the case of constant  $WTE$ <sup>16</sup>. That is,  $\gamma_m(c) = \gamma_m$  and  $\gamma_p(c) = \gamma_p$ . In this case, a high  $WTE$  implies that either the curvature of the cost function is low, or activism is more effective in influencing voter preferences, or both. Then, any increase in the  $WTE$  implies that party activists react more severely to polarization by either candidate, and this precludes them from doing so, even though parties themselves have a widened partisan gap on issues.

**Corollary 1.** *When  $\gamma_m(c) = \gamma_m$  and  $\gamma_p(c) = \gamma_p$ ,*

- i)  $\frac{\partial x^*}{\partial \beta} < 0$  if  $\gamma_p < \frac{1-\gamma_m}{2}$
- ii)  $\frac{\partial x^*}{\partial \beta} < 0$  if  $\gamma_p > \frac{1-\gamma_m}{2}$
- iii)  $\frac{\partial x^*}{\partial \beta} = 0$  if  $\gamma_p = \frac{1-\gamma_m}{2}$ .

Therefore, a greater effectiveness of activism or low marginal costs of participation means that compromise would indeed be chosen by candidates in reaction to greater partisanship. This result helps refocus attention on the role of party structure in electoral campaigns and democratic polities. If parties show a greater willingness to engage with, and persuade, ordinary voters, then even if their own policy

---

<sup>16</sup>For a broad class of power functions of the form  $f(c) = c^\rho$ , the participation aversion and the influence aversion is constant. To see this, notice that  $c \cdot \frac{f''}{f'} = (\rho - 1)$ .

preferences diverge, the fact that there are imminent risks associated with electoral competition would imply that candidates may adopt more moderate stances reflecting the risky nature of campaigns. Parties and activists in democratic states could then provide a natural barrier against polarization as long as they remain actively engaged in the political process.

### An Example

Consider a simple example in which I assume specific functional forms to the influence function and the participation costs. Firstly, for simplicity, let  $\eta = \sigma = 1$ . This is useful in order to understand the theoretical underpinnings of the model more clearly. Further, let participation costs be a power function of the form  $M(c_i) = c_i^\rho$  ( $\rho > 1$ ) and let the influence function be linear,  $P(c_i) = c_i$ . This makes the median voter's utility quasi-linear in activist participation, and  $\eta$  the marginal utility of activist influence.

In this simple parameterization, the equilibrium participation then becomes a solution to the following equation,

$$\begin{aligned}\rho c^{\rho-1} &= \beta \\ \Rightarrow c^* &= \left(\frac{\beta}{\rho}\right)^{\frac{1}{\rho-1}}\end{aligned}$$

The two aversion parameters are then  $\gamma_m = (\rho - 1)$  and  $\gamma_p = 0$ . In the absence of activism,  $c^* = 0$  and  $\bar{x} = \max\{0, \frac{4\alpha-b}{4(\alpha+1)}\}$ . Further, as the demand for activism increases, the candidate platforms decrease and move closer to zero (if  $\bar{x} > 0$ ). Now, the willingness to engage for the activists is given simply by  $WTE = \frac{1}{\gamma_m} = \frac{1}{(\rho-1)}$ . This means that any lowering of  $\rho$ , the elasticity of the marginal cost function, increases the  $WTE$  parameter. In fact the main condition for the result then simplifies to,

$$\frac{\partial x^*}{\partial \beta} < 0 \text{ if } WTE > 1 \text{ or } \rho < 2 \quad (1.4.1)$$

Therefore,  $\frac{\partial x^*}{\partial \beta} < 0$  when  $\rho < 2$ , or the marginal cost function is concave. To understand why this happens, it is important to understand the relationship between marginal costs and benefits of activism. Take the case of activist  $A_R$ . When  $\rho < 2$ , the marginal impact of ideology on participation ( $\frac{\partial c^*}{\partial \beta} = \frac{1}{\beta(\rho-1)} \left(\frac{\beta}{\rho}\right)^{\frac{1}{\rho-1}}$ ) is increasing but convex  $-\frac{\partial^2 c^*}{\partial \beta^2} = \frac{2-\rho}{\rho-1} \frac{1}{\beta} \frac{\partial c^*}{\partial \beta}$  is positive only when  $\rho < 2$ . This has two effects. Firstly, this directly translates into increasing and concave marginal costs of ideological polarization for the activist. Secondly, a convex change in marginal impact of ideology on participation has to be matched by a corresponding increase in marginal benefit, in terms of reduced expected ideological loss. Candidate  $R$  would have to choose platforms such that this trade-off of activist  $R$  is satisfied.

Since activists are ideologically risk averse, platforms that reduce variance (more convergent platforms) provide them with marginal benefits that equate marginal costs of increased polarization. As activists continue to become more polarized, and  $\rho < 2$ , the marginal costs to the activist is concave and therefore, the contributions are increasing in a convex manner. To compensate for this increase, candidate  $R$  chooses a platform closer to ex-ante median ( $\frac{\partial x^*}{\partial \beta} < 0$ ). Simultaneously, candidate  $L$  does the same, and the overall platform polarization reduces, leading to greater convergence.

#### 1.4.2 Other results

**Proposition 4.** *When  $x^* > 0$ , candidate platforms become more extreme if i) candidates' ideological polarization  $\alpha$  increases; ii) benefits of office  $b$  decreases; iii) the demand for activism  $\eta$  decreases; iv) variance in median voter's ideological preference  $\sigma$  increases.*

*Proof.* See Appendix A.1 □

The relationship between equilibrium polarization  $x^*$  and  $\alpha$  or  $b$  has been studied by Bernhardt et al [19], and similar analysis follows in my work. Specifically, polarization increases when candidates' ideal point become more extreme and the benefits of office decreases  $-\frac{\partial x^*}{\partial \alpha} > 0$  and  $\frac{\partial x^*}{\partial b} < 0$ . The relation between platform divergence and  $\eta$  is along expected lines. The rationale is the following. As the demand for activism decreases, it implies that voters weight candidate ideology more heavily compared to activist engagement. This implies activists end up decreasing their participation in the political process. Besides, the increased weight-age on candidate platforms means that the candidates rely less (in marginal terms) on activist participation. This decreased dependency on activists, therefore, translates into more divergent platforms in equilibrium.

As  $\eta$  goes to zero, notice that the equilibrium platform is the same as the case with no activism. That is, as  $\eta \rightarrow 0$ ,  $x^* \rightarrow \bar{x}$ . This can be gleaned by substituting  $\eta = 0$  into the equilibrium equation in Proposition 2. Therefore, as the demand for activism increases in the electoral process, the platforms decrease from  $\bar{x}$  and move towards  $x^* = 0$  (perfect Downsian convergence), meaning there is lesser polarization as the demand for activism increases. Moreover, as risk averse activists engage with the public in order to persuade them to vote for their preferred candidate, it increases participation and wider engagement with the electorate.

Lastly, platform divergence increases when there is greater uncertainty regarding voter's preferences. This makes intuitive sense in that, *ceteris paribus*, candidates in the model are trying to hunt for the median voter's bliss point. Remember that greater uncertainty reduces activist participation because the possibility of more extreme median platforms reduces the marginal benefits for the activists. As this uncertainty or the variance increases, candidates adjust their platform in a way so as to account for this reduced participation from party activists and move more closer to their ideal policy.

## 1.5 Welfare analysis

In this section, I study the welfare effects of activism and polarization on voters and activists. Though I motivated this work on the basis that polarization hurts democratic societies, normative results on voter welfare have shown that this need not be the case for low levels of divergence in platforms (see BDS). This fundamental result is the starting point of my analysis. Consider the welfare of voters first. The welfare of the median voter in a political environment with activists exhibits some interesting properties. When there are activists engaging voters, they generate a positive welfare effect on voters, in addition to the welfare generated from policy stances of candidates. Moreover, since activist participation is independent of the symmetric platform choice of candidates,  $(-x^*, x^*)$ , an increase in polarization does not affect the positive welfare benefits of activism given by  $\eta P(c^*)$ . But, the presence of this positive utility from activism implies that any increase in the exogenous parameters  $(\beta, \eta, \sigma)$  that affect participation also impacts the welfare of voters by shifting this positive utility from activism.

Let  $W_{mv}(-x, x)$  be the welfare of the median voter under the symmetric equilibrium platforms of the two candidates. Then,

$$W_{mv}(-x, x) = - \int_{-\sigma}^0 (x + \mu)^2 d\mu - \int_0^{\sigma} (x - \mu)^2 d\mu + \eta P(c^*)$$

**Lemma 4.** *If  $x > 0$ , then welfare of median voter is such that, i)  $W_{mv}(-x, x) > W_{mv}(0, 0)$ ; ii) it is maximized at  $x^{vo} = \frac{\sigma}{2}$ . Moreover,  $W_{mv}(-\frac{\sigma}{2}, \frac{\sigma}{2}) = \eta P(c^*) - \frac{\sigma^3}{6}$ .*

*Proof.* See Appendix A.1 □

The welfare of party activists provides for a contrasting scenario. Let  $W_{act}(-x, x)$  be the joint welfare of the two party activists under the symmetric equilibrium platforms. Then,

$$W_{act}(-x, x) = 2 [K - x^2 - \beta^2 - M(c^*)]$$

Notice that the activists' welfare is maximized when there is Downsian convergence. That is, irrespective of how large the partisanship between the two activists is, paradoxically, they prefer perfect Downsian convergence on the ex-ante median. However, then, the addition of activists alters the socially optimum welfare since on the one hand voters prefer a small level of divergence  $(-\frac{\sigma}{2}, \frac{\sigma}{2})$ , and on the other, activists' optimum is at  $(0, 0)$ . The overall social welfare then reflects this clash of preferences between activists and voters.

To see this more clearly, suppose there were a social planner who maximizes the combined welfare of activists and voters. Let  $W_{tot}(-x, x)$ <sup>17</sup> be the total social welfare of the electoral process. This provides us a intuitive way of thinking about parties as an important political institution, and to look at rationale of normal citizens to turn to activism. Welfare results could then address some very pertinent questions regarding implementing socially optimal platforms, role of party polarization and the design of optimal institutions to improve the total social welfare. Before proceeding, I will make the following assumption on the value of  $K$ , the benefits of party organization.

**Assumption 1:**  $K$  is such that  $W_{mv}(0, 0) < W_{act}(0, 0) < W_{mv}(0, 0) + \sigma^2$

The assumption on  $K$  merely ensures that at the activists' optimum  $x^{ao} = 0$ , their welfare is higher than that of voters. If this condition failed, activists would strictly be worse off for any level of polarization compared to voters<sup>18</sup>. Now, given this assumption and the formulation of overall welfare, I can write  $W_{tot}(-x, x)$  as the following,

---

<sup>17</sup>I will refer to the welfare functions  $W_i(-x, x)$  as  $W_i^x$  where possible for reasons of exposition.

<sup>18</sup>Though this may not be ruled out in democratic countries, it nevertheless ceases to be an interesting case, in the context of answering some important questions regarding the relevance of parties as political institutions.



$$W_{tot}(-x, x) = W_{act}(-x, x) + W_{mv}(-x, x)$$

**Lemma 5.** *The socially optimal level of polarization is  $x^{so} = \frac{\sigma}{(\sigma+1)} \cdot \frac{\sigma}{2} < x^{vo}$ . Every symmetric equilibrium  $(-x, x)$  where  $x \in [0, \frac{\sigma}{2}]$  is pareto efficient. A social planner can always select a  $b^*$  or  $\eta^*$  such that  $x^{so}$  is chosen in equilibrium.*

*Proof.* See Appendix A.1 □

The analysis from here on will concentrate on the pareto efficient equilibria, meaning the set of equilibrium outcomes in  $[0, \frac{\sigma}{2}]$ . Notice the departure from the analysis of BDS. In the presence of parties as political institutions, the overall welfare is optimized at a lower level of polarization. This moderation is driven by the presence of risk-averse activists who expend costly effort in the political competition process. The lesser polarization optimum is, in a way, a compensation for the costly effort decision of activists.

However, the socially optimal level need not result as an equilibrium of the political competition process. To implement this socially optimal equilibrium, the planner can adopt two instruments of policy. The first, benefits of office  $b$ , has been proved and studied by BDS. The second is of more interest for this paper. It says that the social planner can create conditions such that the demand for activism is high (or low), depending on whether the equilibrium polarization is above or below  $x^{so}$ .

It is then interesting, given the relationship between activist polarization and candidate platforms, to study the impact of partisanship on overall welfare for any equilibrium level of polarization  $x \in (0, x^{so})$ . As we observed earlier, when the level of participation in activism is high (condition  $i$ ) of Proposition 3), the equilibrium level of polarization decreases. Besides, total welfare is increasing in the interval  $[0, x^{so})$ , and coupled with high  $WTE$ , this implies that partisanship in fact reduces overall welfare.

**Proposition 5.** *Given an equilibrium level of polarization  $x \in (0, x^{so})$ , an increase in the partisan gap between activists reduces total welfare when activists have a high WTE and improves welfare when WTE is low; and vice versa for  $x \in (x^{so}, x^{vo}]$ .*

*Proof.* See Appendix A.1 □

This result is a bit counter-intuitive, but follows from the first result of Proposition 3. The implication of this result is that increased partisanship calls for lowering the WTE of activists in order to curb their participation in the electoral process. This *lowering of willingness to engage* could be seen as introducing barriers to participation in the campaign. If this is not so, then a higher willingness combined with greater partisanship reduces polarization of platforms in equilibrium. By introducing barriers to participation (higher costs, say), the social planner can implement an equilibrium that provides more platform separation that is welfare improving.

The reason why this result emerges is that when willingness to participate is high, the compensation to activists for their costly efforts happens through lower polarization. However, since platforms were below socially optimal level to start with, this further hurts voters and reduces overall welfare. On the other hand, when polarization exceeds the social optimal, increased partisanship combined with high WTE improves welfare.

### 1.5.1 Rationale for becoming an Activist

I will now look at the problem considered by Aldrich, pertaining to the participation decision of activists. Aldrich, in an intuitive way, characterized the decision to join a party by constructing a "calculus of participation" for individual citizens. Though I have not explicitly concentrated on this aspect in my paper so far, the welfare characterization I have developed above is useful in addressing the question of benefits of party association.

Before answering this question, I will impose a few restrictions on the size of benefits from party association. This is done to ensure that analysis of equilibrium polarization could be restricted to the pareto set of outcomes.

**Definition 1.** Let  $\bar{K} = \frac{1}{2}[\eta P(c^*) + 2M(c^*)] + [\beta^2 - \frac{\sigma^2(3-\sigma)}{8}]$ .

*Assumption 2:*  $K < \bar{K}$

Given this formulation, the following proposition establishes a welfare-indifference result.

**Proposition 6.** *Under Assumptions 1 and 2, there exists a unique no-arbitrage equilibrium  $(-x^{na}, x^{na})$  in which both the median voter and activists achieve the same welfare. This equilibrium is decreasing in the level of partisanship  $\beta$ . Moreover, if the equilibrium level of polarization is above  $x^{na}$ , there is a surplus associated with being a voter, and for equilibrium polarization below  $x^{na}$ , there is a surplus enjoyed by party activists.*

*Proof.* See Appendix A.1 □

The existence of a no-arbitrage equilibrium is indeed useful for two reasons. Firstly, it clearly points at exactly which of the two actors – activist or the voter – benefit from the political process. Given the political environment, the level of candidate polarization determines which of the two enjoy a surplus. Clearly, if the polarization is greater than  $x^{na}$ , the fact that the welfare of an activist is decreasing in  $x$  and that of the voter is increasing in  $x$  (in the pareto set  $[0, \frac{\sigma}{2}]$ ) ensures that the voter enjoys this surplus associated with greater polarization. The opposite holds for  $x < x^{na}$ . Secondly, the no-arbitrage equilibrium is decreasing in the extent of partisanship among activists. This is evident by noting that any increase in activist polarization shifts the welfare curve of activists down and that of voters, up. This, by the single crossing property of the curves (Assumption 1), implies that the no-arbitrage equilibrium shifts to the left.

**Corollary 2.** *For any equilibrium  $x \in (x^{na}, x^{so}]$ , the voter surplus is positive and increasing in  $x$ . Similarly, for any equilibrium  $x < x^{na}$ , the activist surplus is positive and decreasing in  $x$ .*

Why is the no-arbitrage equilibrium of any interest? There are two reasons. Firstly, the presence of this surplus provides a rationale for participating as an activist. Whenever the surplus is positive for activists, it implies that the benefits from associating with a party and engaging with the public, though costly, provides a higher welfare than being a voter. This could be interpreted as an useful condition for motivating participation in activism. A decision to become an activist with a party could then be viewed as one based on welfare implications of doing so.

Secondly, since the social planner can choose a level of  $b$  or  $\eta$  to implement any level of equilibrium polarization, the same applies to the no-arbitrage equilibrium. Suppose the planner's interest is to maintain welfare indifference between the activists and voter. Then, the level of overall welfare associated with the no-arbitrage equilibrium depends on the extent of partisanship in the society.

**Corollary 3.** *The following holds:*

- i) If  $x^{na} < x^{so}$ , then  $\frac{dW_{tot}(-x^{na}, x^{na})}{d\beta} < 0$ ,  $\frac{dW_{tot}(-x^{na}, x^{na})}{d\eta} < 0$ ,  $\frac{dW_{tot}(-x^{na}, x^{na})}{d\sigma} > 0$ .
- ii) If  $x^{na} > x^{so}$ , then  $\frac{dW_{tot}(-x^{na}, x^{na})}{d\beta} > 0$ ,  $\frac{dW_{tot}(-x^{na}, x^{na})}{d\eta} > 0$ ,  $\frac{dW_{tot}(-x^{na}, x^{na})}{d\sigma} < 0$ .

Clearly, the benefits of choosing the no-arbitrage equilibrium depends on whether it is below or above the social optimum  $x^{so}$ . The implications of this is that when partisanship between parties increase, it may be beneficial for the society as a whole to have a level of polarization  $x > x^{na}$ , meaning that providing voters a welfare surplus also increases overall welfare. Of course, in this case, activists face a welfare cost (which is the voter surplus) of being associated with political party. This brings us to the question of how to incentivize party association in order to mitigate this costs of associating with activism.

### 1.5.2 Strength of party organization

Given the framework of my previous analysis, in this subsection, I will try to reconcile the ideas of achieving the social optimum polarization on one hand, and providing the right incentives for people to associate with party organizations. Though there are potential benefits pertaining to the "civic engagement" role of party activists, this has to be balanced and reconciled with the broader voter welfare. The results presented in the previous subsection provides an intuitive way to reconcile these opposing interests. In particular, strengthening party organizations (an increase in  $K$ ) could potentially provide a tool to achieve an equilibrium in which the society reaches a socially optimal level of platform polarization that also makes the participants in the democratic process indifferent in welfare terms. The following proposition formalizes this idea.

**Proposition 7.** *For any given political environment  $E = (\alpha, \beta, \eta, \sigma, b)$ , there exists a  $K^*$  such that when  $K = K^*$ , implementing the socially optimal level of polarization  $(-x^{so}, x^{so})$  makes both the voters and the activists welfare-indifferent, i.e.,  $W_{mv}^{x^{so}} = W_{act}^{x^{so}}$ . Moreover,  $K^*$  is increasing in the level of party polarization and the extent of demand for party activism.*

*Proof.* See Appendix A.1 □

The above proposition provides a possible policy tool that may strengthen (or weaken) party organization in democratic countries. Strengthening parties has the benefit that it wrests the balance between activists and the general public in the sense that it erodes the voters' surplus when trying to implement a socially optimal level of polarization. However, as the analysis suggests, the opposite is true when the benefits of party association is very high. In this case, it may be required to design policies that may weaken party structures so that the social optimum makes them indifferent, in welfare terms, to the voters.

**Corollary 4.** *If  $K < K^*$ , the social optimum level of polarization provides a welfare surplus to voters. On the other hand, when  $K > K^*$ , the social optimum level of polarization provides a welfare surplus to party activists.*

These results shows us the trade-offs associated with designing political institutions. Suppose a fraction of the public decide to become associated with activism (costly mode of participation) and the rest of them remain voters (non-costly method of participation). Then it is very important that parties as democratic institutions have the right level of "strength" compared to the other institutional arrangements in the society – namely,  $b, \eta$ . Party strength could be used as an important policy tool to ensure that the democratic dividend is equally distributed between activists and voters. As the level of partisanship in society rises, parties can act as an useful institution for moderation. Moreover, by carefully choosing a level of  $K$ , the social planner could achieve both welfare maximum and welfare-indifference for the participants of a democracy.

## 1.6 Extensions

### 1.6.1 Activism in Noisy campaigns

Suppose the policy platform of candidates are observed with noise by median voter, and activists' role is to inform the median voter of the precise position. If  $X_i$  is the true position of the candidate, the policy observed by the median voter is  $\tilde{X}_i = X_i + \eta_i$ , where  $\eta_i$  is a random variable (noise term) with expectation zero and variance  $\sigma_i^2$ . Further, contribution from activists reduces the variance of the noise term. If  $c_i$  is the contribution from the activist, then  $\sigma_i^2 = a(c_i)$ . The following assumptions are made on the functional form of  $a(\cdot)$ <sup>19</sup>:  $a'(\cdot) < 0$ ,  $a''(\cdot) > 0$ ,  $a'''(\cdot) < 0$  and  $a(0) > 0$ . First two conditions ensure that as activists contribute more, the

---

<sup>19</sup>We additionally assume that the noise reduction mechanism  $a(\cdot)$ , is the same for both candidates.

variance of noise function is decreasing, and convex. The subsequent condition implies that the concavity of marginal variance, and the last condition states that, in the absence of activism, there is a positive level of noise in platforms, meaning voters imperfectly observe platform of candidates.

This formulation naturally implies that greater activist participation is beneficial for candidates since it reduces the variance of the platforms, and since the voter is risk-averse, less variance is preferred. Activists or volunteers, then, have an important role in conveying - through door-to-door canvassing or phone calls - the true policy stance of their candidate.

Before presenting the results, it is important to glean the role of noise reduction function. Remember, activism is now not a persuasive tool, but restricted to only reducing the variance of the noisy platform. I introduce the parameter  $\gamma_n(c)$  to define the efficiency of activism in reducing the noisiness of platforms. That is, Efficiency of Marginal Noise Reduction,  $\gamma_n(c) = -c \cdot \frac{a''}{a'}$ . As in the baseline model, the willingness to engage is defined by  $WTE_N(c) = \frac{1}{\gamma_m(c) + \gamma_n(c)}$ . For the sake of simpler exposition, let  $WTE_N$  be independent of  $c$  ( $\gamma_m(c) = \gamma_m$  and  $\gamma_n(c) = \gamma_n$ ).

**Proposition 8.** *In a noisy electoral campaign with activism, there exists an unique symmetric equilibrium in candidate platforms. Furthermore,*

$$i. \frac{\partial x}{\partial \beta} < 0 \text{ if } \gamma_n < \frac{1-\gamma_m}{2}; \text{ ii. } \frac{\partial x}{\partial \beta} > 0 \text{ if } \gamma_n > \frac{1-\gamma_m}{2}$$

*Proof.* See Appendix A.2 □

Proposition 8 shows that the main equilibrium and comparative statics result holds. The details of the same are confined to Appendix B.

### 1.6.2 The role of soft money

Though the reliance of grassroots activists is an important avenue of campaigning, the role of soft money in the form of PACs or super PACs also have been playing an increasingly important role. McCarty, Poole and Rosenthal [50], for e.g., find that

large contributions and contributors on average were more extreme<sup>20</sup> (on either side of the political spectrum), and moreover, they tended to favor extreme candidates. In some way, this type of soft money contributions act as a substitutable good to activism. Therefore, when candidates have access to big money, their reliance on the party organization and its activists goes down. This in turn may provoke candidates to polarize away from each other, and towards more extreme ideologies.

To see this mechanism, I will modify the model to consider the role of big money<sup>21</sup>. Suppose  $S$  is the the available soft money contributions for either candidate. Then, the candidates have two goods that are employed for influence in elections – activist participation  $c_i$  and soft money  $S$ . Moreover, I modify the influence function to include soft money parameter. That is,  $P(S, c_i)$  is the total influence generated by campaigning, such that  $P_2(.) > 0$ ,  $P_{22}(.) < 0$ ,  $P_1(.) > 0$ ,  $P_{11}(.) < 0$ , and  $P_{12}(.) < 0$ . The concavity assumption holds as before, and the last assumption states that  $S$  and  $c_i$  are strategic substitutes. This makes intuitive sense. Candidates need money to spend on advertisements, hiring campaign staff, on personalized communication to voters, and so on. Therefore, money supplements the traditional grassroots campaign of activists.

What this suggests is that the presence of big soft money may *crowd out* the role of activism, thereby decreasing the level of participation. This crowding out of activists may then provide candidates to polarize for much the same reasons as discussed earlier. Any decrease in the marginal effect of activism reduces the candidates' incentive to compromise. This leads to greater polarization in equilibrium.

**Lemma 6.** *In the presence of soft money in campaigns, the equilibrium polarization and participation are such that, i)  $\frac{dc^*}{dS} < 0$  and ii)  $\frac{dx^*}{dS} > 0$ .*

*Proof.* See Appendix A.2 □

---

<sup>20</sup>Please refer to chapter 5 of MPR.

<sup>21</sup>For the sake of exposition, I will abstract away from strategic interaction of big donors. I will exogenously assume that candidates have a share of soft money contributions, and then use this to study its impact on polarization and activism.



Lemma 3 shows how soft money has a crowding out effect on activism and pushes platforms to more extremes as a result. The fact that this kind of soft money reaches more extreme candidates in the first place would only exacerbate its effect on political polarization ( $\frac{dx^*}{dS}, \frac{dx^*}{d\alpha} > 0$ ). As a result, this suggests a rationale for curbing this kind of big money spending by individual contributors or organized groups<sup>22</sup>. In fact, by limiting the campaign expenditure (public funding of elections, for e.g.), it is possible to incentivize political candidates to rely more heavily on traditional grassroots campaigning involving activists. My analysis suggests that this kind of reform could be used as an useful policy instrument. Curbing the use of soft money and increasing the dependence of activists could help counter balance the current trend of excessive role of big money in the political process.

## 1.7 Conclusion

I have analyzed a model of activism to address the question of whether, and how, the participation of a more polarized electorate in the electoral process affects political polarization.

The main finding of my model is that when activists become more polarized, and their willingness to engage is above a threshold, the equilibrium platforms of candidates tend to converge, or in other words, political polarization decreases. The implication of this result is that electorate polarization, on its own, is insufficient to explain political polarization. The combination of activist partisanship and the willingness to engage in the political process together determine the extent of political polarization. Further, I find that polarization reduces as the demand for activism increases or when the uncertainty around median voter's ideal policy is lower.

Some important welfare implications emerge from my analysis. Specifically, I establish that when the willingness to engage of activists is high and for moderate

---

<sup>22</sup>The so called "527 group" spending, for e.g., places no upper bounds on how much and who to contribute. Some prominent ones include Club for Growth, MoveOn.org, New Democrat Network, among others.

levels of polarization, overall welfare decreases as partisanship increases. This means that societies with greater democratic values may indeed provide lesser choice to the voters, and therefore, make them worse off in welfare terms.

Moreover, I find that in any symmetric equilibrium, either the activists or the voters get a welfare surplus. This surplus depends on the extent of polarization and benefits from party association. I also establish the presence of a unique no-arbitrage equilibrium in which activists' and voters' welfare are equivalent. Any level of polarization below this threshold of no-arbitrage provides the activists with a welfare surplus, and vice-versa.

Finally, I look at the possibility of implementing the socially optimal level of polarization. Doing so, by changing the benefits of office or demand for activism, creates a welfare surplus either for the activist or the voters. I characterize the benefit from party association that guarantees welfare indifference between the activist and voter at the socially optimal level of polarization. Further, I find that this benefit from party association is increasing in extent of partisanship and the demand for activism.

## Chapter 2

# Cheap Talk and Strategic Substitutability

### 2.1 Introduction

Crawford and Sobel [26] (hereafter CS), in their seminal paper, consider the problem of strategic communication between an informed Sender and uninformed Receiver. The main result of their work is that when the Sender is biased, cheap talk messages -costless, unverifiable and non-binding- never perfectly reveal information and communication is always noisy. Equilibrium communication takes the form of (noisy) state partitions. The Sender reveals that the state is within a certain partition, but never truthfully reports the true state, as long as there is some degree of conflict of interest. This fundamental analysis of cheap talk has spurred a large literature on both the theoretical<sup>1</sup> and applied<sup>2</sup> front.

---

<sup>1</sup>Farrell and Gibbons [28] study the case of single sender and two receivers; Krishna and Morgan [47], and Li et al [48] analyze information transmission between multiple senders and a single receiver; Galeotti et al [33] look into cheap talk in networks; Battaglini [17] models the case of many senders with a multidimensional state space; Sobel [67], Morris [54] investigate the role of reputation building in a repeated cheap talk setting; and Baliga and Morris [14] throw light on coordination incentives and cheap talk communication.

<sup>2</sup>Cheap talk communication has been used extensively to study pertinent questions in a wide array of fields including political science (see Austen-Smith [11] [12], Gilligan and Krehbiel [35], and Morgan and Stocken [53]), organizational theory (see Melumad and Shibano [51], Rantakari

An important feature of the CS framework is that the Receiver is the only decision maker. In this paper, I focus on a variation of the standard set-up. Instead of a single decision maker, I consider a scenario in which both the Sender and Receiver take an action after communication, and further, these actions are substitutable. When players' actions are strategic substitutes, a message from the Sender potentially affects the subsequent actions of both the Sender and Receiver. This interaction between communication and (ex-post) actions change the incentives for information revelation, and hence, the nature of communication equilibria.

To see this, take the original CS model. Reporting the true state is never optimal for the Sender since the Receiver's action is always biased downwards. However, when the Sender is allowed to take an action post the communication round, the Sender can anticipate the (posterior) beliefs induced by her message to the Receiver and precisely predict the Receiver's action, and thereby, best respond to this action so as to maximize her own payoff. This implies that what matters for truthful communication is whether the Sender, given the permissible set of actions, is able to compensate sufficiently for the Receiver's action that is induced by the message.

The addition of action substitutability, therefore, alters the insights of the standard CS model by allowing the Sender to reveal the state truthfully. The amount of information conveyed crucially depends on whether the Sender is able to communicate the lowest state, and the highest state, truthfully. When the former is violated, the Sender does not have any incentive to reveal her private information, and there is communication breakdown in the sense that only a '*babbling*' equilibrium of the cheap-talk game exists. On the other hand, when the Sender can reveal both the lowest and highest state, then, irrespective of the conflict of interest, full information revelation is achieved. Therefore, if the domain of the Sender's action set is sufficiently large enough, the Sender can credibly reveal the state and subsequently take an action that precisely compensates for the Receiver's action.

---

[64], and Alonso et al [5]), finance (see Morgan and Stocken [52]) and macroeconomics (see Stein [68] and Moscarini [56]), among others.

When the Sender is able to reveal the lowest state but is unable to do so for the highest state, communication deteriorates and there is only partial revelation of information. A partial revelation equilibrium is a cut-off equilibrium, in which the Sender reveals truthfully only upto a threshold state, and beyond this threshold, pools information. I characterize the most informational equilibrium as the one with the highest threshold. Comparative statics reveal that this threshold is higher when biases are closer to each other, and when the upper bound on actions is greater. The former result is similar to the original CS model argument, while the latter is a direct consequence of action substitutability. A greater upper bound on the action set implies the (upward biased) Sender is able to compensate efficiently for a greater measure of types. This enables more truthful communication, leading to a higher informational threshold. Hence, either a decrease in conflict of interest between players, fixing the bound, or an increase in the domain of available actions, keeping the bias constant, leads to more information transmission.

Next, I study some welfare properties of partial revelation equilibria. Intuitively, I find that the most informative equilibrium (ex ante) Pareto dominates every other partial revelation equilibria. Further, contrary to the nature of CS equilibria, I establish that the most informational equilibrium also achieves interim efficiency<sup>3</sup>. That is, for every type of Sender, the most informational equilibrium is at least weakly preferred. There are two reasons for this finding. First, a higher threshold reduces the measure of states for which the Sender is unable to compensate sufficiently. Second, by providing more information, the Sender raises the expected action of the Receiver over the states that are not revealed truthfully. These two effects reinforce each other making every Sender type at least weakly better off from revealing more information.

Almost all of the work on cheap talk models so far have neglected the possi-

---

<sup>3</sup>In the most informational partition of the standard CS setup, there is always a low type Sender that prefers the babbling equilibrium over the more informational partition equilibrium. Hence, even though partitional equilibria could be ordered in the Pareto sense, they never achieve never interim efficiency. See Chen et al. [23] for more on this point.

bility of information transmission with action substitutability<sup>4</sup>. However, a number of real world situations involve this kind of interdependence. For example, leaders of countries working cooperatively to achieve common international policy objectives (military intervention or intelligence gathering, say), or organisations with multiple teams working together on a common project, or different departments within a government trying to jointly implement a policy that requires varying effort decisions, are all instances in which there is an element of information sharing and substitutability in actions. This paper provides an useful starting point to study the nature of strategic information transmission in such interdependent environments.

The papers closely related to my work are the ones by Kartik et al [46], Kartik [45], Morgan and Stocken [52], and Ottaviani and Squintani [60]. Kartik et al [46] derive a fully separating equilibrium with lying costs and the possibility of a naive Receiver. The key condition driving their result is the unboundedness of the domain of private information. The full information revelation result I characterize relies instead on the domain of permissible actions for the Sender. As long as the available domain is large enough so that the Sender is able to credibly reveal both the lowest type and highest type information, full revelation ensues.

In Kartik [45], truthful communication is restricted by the presence of a bound on the state space, leading to incomplete separation. Morgan and Stocken's [52] threshold equilibria result is driven by uncertainty in the extent of bias of the informed party. Finally, Ottaviani and Squintani [60] construct cheap talk equilibria with naive receivers and a bounded state space in which communication is truthful upto a threshold, and partial beyond. The key difference of my partial revelation result is that it is driven by substitutability and restricted domain of actions (a form of resource constraint). In this paper, resource constraints indirectly affect the capacity of the Sender to compensate, and this in turn affects truthful communication

---

<sup>4</sup>Barring Alonso [4], who considers a principal-agent setting in which an uninformed principal controls decision rights and the informed agent communicates information strategically, and actions of the two players are either strategic complements or substitutes.

in equilibrium.

The paper proceeds as follows. In Section 2.2, I present a simple example to show the main intuition driving my results. Section 2.3 builds the basic model and develops the necessary condition for full information revelation equilibrium. Section 2.4 contains the results pertaining to partial information revelation equilibria. In Section 2.5, I present some comparative statics results and welfare analysis of the partial revelation equilibrium follows in Section 2.6. Finally, Section 2.7 provides concluding remarks.

## 2.2 Leading Example

Consider a variant of the basic Crawford-Sobel set-up with strategic interdependency in actions. An informed player, say  $S$ , receives a perfectly observable signal about the state of the world  $\theta$ , drawn from an uniform distribution  $[0, 1]$  and communicates this information through a cheap talk message  $m$  to an uninformed player, say  $R$ . Upon communication, both  $R$  and  $S$  take actions in a way that affects both their payoffs. Let the modified utility function be the following:

$$U^R = - \left[ \left( \frac{x_R + \eta x_S}{1 + \eta} \right) - \theta \right]^2$$

$$U^S = - \left[ \left( \frac{x_S + \eta x_R}{1 + \eta} \right) - \theta - b \right]^2$$

Observe the small departure from the CS set-up. Both players now are allowed to take actions after communication, and actions are substitutes in that  $\frac{\partial^2 U^i}{\partial x_R \partial x_S} < 0$ , where  $\eta \in (0, 1)$  captures the degree of substitutability<sup>5</sup>. Further, let the actions of players  $x_i$  have a domain  $[-a, a]$ . Given this structure, when  $S$  truthfully reveals the true state of the world through her message,  $m = \theta$ , the two players solve the following best responses:

$$R: x_R = (1 + \eta)m - \eta x_S$$

$$S: x_S = (1 + \eta)(m + b) - \eta x_R$$

---

<sup>5</sup>The case of  $\eta = 1$  is one where actions are perfect substitutes. In fact under perfect substitutes, the incentives I consider are not valid and equilibrium existence is not guaranteed. I will discuss this, and more, later.

To simplify the exposition, let  $b = \frac{2}{5}$  and  $\eta = \frac{1}{2}$ . Equilibrium actions after (truthful) messaging are given by:  $x_R^* = m - \frac{2}{5}$ ,  $x_S^* = m + \frac{4}{5}$ . Notice immediately that full information revelation is possible if  $a \geq \frac{9}{5}$ . This is so because  $S$  is able to compensate precisely even for the highest type,  $\theta = 1$ . On the other extreme, if  $a < \frac{4}{5}$ , no information can be credibly revealed by  $S$ , since irrespective of what the true state is, reporting the truth is never optimal for  $S$ . This stems from her inability to sufficiently compensate even for the lowest type signal. For example, when  $a = \frac{2}{5}$ , the equilibrium action of the sender under truthful communication is  $x_S^* = \frac{2}{5}$ , irrespective of the state. However,  $S$  can inflate her signal in order to make  $R$  play a higher action. To see this, suppose instead of  $m = 0$ ,  $S$  inflates and sends a message  $m = \frac{4}{5}$ . Then,  $R$  best responds by taking an action  $x_R^* = \frac{2}{5}$ . But notice that  $S$  can fully anticipate this response by  $R$  and suitably adjust her optimal action accordingly. In particular,  $S$  takes an action  $x_S^* = \frac{3}{5} - \frac{1}{5} = \frac{2}{5}$ . Though  $S$ 's action has not changed, she has managed to push  $R$ 's action upwards, and thereby achieves a payoff of 0. But this incentive to misrepresent means that  $R$  would never believe any message from  $S$ , and therefore communication is rendered ineffective in equilibrium.

Conversely, when  $\frac{4}{5} < a < \frac{9}{5}$ ,  $S$  has an incentive to reveal some information. To see this, let  $c = 1$ . Then, for any  $\theta \in [0, \frac{1}{5}]$ ,  $S$  reveals the state truthfully since her optimal action is within the domain of available actions (in this case  $x_S^*(\frac{1}{5}) = 1$ ). But, for any  $\theta > \frac{1}{5}$ ,  $S$  cannot sustain a truthful messaging strategy. To see this, suppose  $\theta > \frac{1}{5}$ , and  $S$  reports truthfully. Then the optimal action for  $S$  is bounded by  $x_S = 1$ , while  $R$  provides the residual as demanded by her best response function, which is  $x_R = \frac{3}{2}\theta - \frac{1}{2}$ . This cannot be an equilibrium since there is underprovision as far as  $S$  is concerned:  $S$  gets a payoff of  $U_S = -\left(\frac{1+\frac{1}{2}(\frac{3}{2}\theta - \frac{1}{2})}{\frac{3}{2}} - \theta - \frac{2}{5}\right)^2 = -(\frac{1}{10} - \frac{\theta}{2})^2 < 0$  for  $\theta > \frac{1}{5}$ . Therefore,  $S$  has an incentive to exaggerate her information in order to induce  $R$  to contribute more. This precludes separation beyond  $\theta = \frac{1}{5}$ .

In fact, all types above this cutoff must pool and send the highest message,



$m = 1$ . This is primarily because the signals are (imperfectly) invertible in this environment. Any pure message  $m < 1$  could be interpreted as coming from one of the many possible (weakly lower) types. For instance, when  $\theta = \frac{2}{5}$ ,  $S$  would want to exaggerate and send a message of at least  $m \geq \frac{3}{5}$ , since this would ensure that  $S$ 's action is within the bound  $a = 1$ . Say  $S$  sends  $m = \frac{3}{5}$ . But this message could possibly come from any of the types  $\theta \in (\frac{1}{5}, \frac{2}{5}]$ , each of whom have incentives to deviate and send  $m = \frac{3}{5}$ . Hence,  $R$  could rationally invert this message and form beliefs accordingly<sup>6</sup>. But if this is the case, every type in the interval  $(\frac{1}{5}, 1]$  would find it profitable to send the highest pooling message possible,  $m = 1$ . At most, there is a partial revelation equilibrium, in which  $S$  reveals truthfully (or separates) for  $\theta \in [0, \frac{1}{5}]$  and pools her messages for  $\theta \in (\frac{1}{5}, 1]$ .

The example suggests a novel trade-off for information transmission with action substitutability. That is, the ability to truthfully reveal information depends on how large the bounds on actions are, namely extent of  $a$ , on  $S$ . Crucially, as  $a$  increases (the interval  $(\frac{4}{5}, \frac{9}{5})$  in the example above), there is more information revealed by  $S$ . As the upper bound on the available domain of actions expands,  $S$  is able to best respond to her message and expected action of  $R$ . This way, the informed player is able to provide more information, regardless of the extent of the biases between the two players.

## 2.3 The Model

Consider two players, a receiver  $R$  and sender  $S$ , who decide on contributions to a joint project. The payoffs from the project is contingent on an unknown state  $\theta \in \Theta \equiv [0, 1]$ , distributed according to the density function  $f(\cdot)$ . The sender

---

<sup>6</sup>For precisely a similar argument, partition equilibrium of the kind developed by CS are also ruled out on the interval  $(\frac{1}{5}, 1]$ . Again, this is so because there would not exist an indifference type in this interval, since there is a natural propensity to inflate information. This incentive to exaggerate ensures that if there are two partitions, the high types in the lower partition would find it profitable to deviate to the higher partition, precluding the existence of an indifference type in the first place.

receives a perfectly observable private signal about the state  $\theta$ , while the receiver has no information. Each player's utility is given by  $U(\phi^i(x_i, x_{-i}), \theta, b_i)$ , where  $\phi^i(\cdot)$  is the player-specific (symmetric) joint contribution function<sup>7</sup>. The contribution function depends on a player's own action  $x_i$ , as well the contribution made by the other player,  $x_{-i}$ . Actions of players are such that  $x_i \in V \subseteq \mathbb{R}$ , where the set  $V$  is closed and compact. The bias parameter  $b_i$  measures the conflict of interest between the two players. The standard Crawford-Sobel assumptions on the utility function of players hold. Specifically,  $U(\cdot)$  is twice continuously differentiable,  $U_1(\cdot) = 0$  for some  $\phi^i$ ,  $U_{12}(\cdot) > 0$ ,  $U_{13}(\cdot) > 0$  and  $U_{11}(\cdot) < 0$  so that  $U$  has a unique maximum for any given pair  $(\theta, b_i)$ . This implies that there is an unique joint contribution function  $\phi^i$  for each player that satisfies their maximisation problem.

The utility functions of the players satisfies the condition  $\frac{\partial^2 U}{\partial x_i \partial x_j} < 0$ , meaning that actions of the two players are strategic substitutes. For sake of exposition, I normalize the bias of receiver to 0 and that of sender to  $b > 0$ <sup>8</sup>. The two players therefore maximize their payoff functions given by  $U(\phi^R(x_R, x_S), \theta)$  and  $U(\phi^S(x_S, x_R), \theta, b)$ , respectively. Players have action interdependence in the sense that each players' action  $x_i$  affects the contribution function of the other player  $\phi^{-i}(\cdot)$ , and hence their action,  $x_{-i}$ . Since  $b > 0$  and  $U_{13}(\cdot) > 0$ , it implies that  $\phi^S(\cdot) > \phi^R(\cdot)$  for every  $\theta$ . I make the following further assumptions on the functional form of  $\phi^i(\cdot)$  to ensure interior solution to the contribution decision of the players:

**Assumption 1:** *Non-decreasing marginal contribution* -  $\frac{\partial \phi^i(\cdot)}{\partial x_i} \geq 0$

**Assumption 2:** *Non-negative spillover* -  $\forall i, j \neq i : \frac{\partial \phi^i(\cdot)}{\partial x_j} \geq 0$

**Assumption 3:** *Imperfect substitutability* -  $\forall i, j \neq i : \frac{\partial \phi^i(\cdot)}{\partial x_i} > \frac{\partial \phi^i(\cdot)}{\partial x_j}$  and  $\frac{\partial \phi^i(\cdot)}{\partial x_j} < 1$

---

<sup>7</sup>The assumption of symmetry is not important in order to generate the main findings. It, however, aides comprehension of the same.

<sup>8</sup>The biases could be a function of  $\theta$  in that  $b(\theta)$  may be the extent of conflict of interest, instead of a constant  $b$ . This, however, does not change the main results of the paper as long as single-crossing property holds, meaning  $U_{13} > 0$ .

Assumption 1 ensures that the contribution function is non-decreasing in the player's own action, while the second assumption implies that a player's contribution function is non-decreasing in the other players' action. Assumption 3 implies that the marginal contribution effect dominates the spillover effect. It also rules out perfectly substitutable actions, since when actions are perfect substitutes, there is no guarantee of an interior equilibrium (actions are pulled apart to  $-\infty$  or  $+\infty$ , depending on the biases of the players)<sup>9</sup>.

The game proceeds in two stages. In the first stage, the sender observes the true state  $\theta$  and sends a message (or report)  $m \in M$  to the receiver. Let this messaging strategy be defined by a mapping from the private signal of sender into a message  $m = \mu(\theta)$ . Let  $p(\theta \mid m)$  denote receiver's posterior beliefs on  $\theta$  after receiving the message  $m$ . In the next stage, both players simultaneously decide on contributions.

A communication equilibrium of this game is a (pure strategy) perfect Bayesian Equilibrium (PBE) which satisfies the following properties:

- given the sender's message  $m$  and the posterior beliefs  $p(\cdot \mid m)$  over the state, both players simultaneously choose actions that maximizes their respective payoffs
- the posterior beliefs are updated using Bayes' rule where possible
- given the beliefs and second stage contributions  $x_R(m)$  and  $x_S(\theta, m)$ , the sender's reporting strategy  $\mu(\cdot)$  maximizes the expected payoff in the first stage

A PBE always exists in games with cheap talk. This is a babbling equilibrium in which sender's message is ignored and receiver acts based on her prior information,

---

<sup>9</sup>Take the example presented in Section 2 and substitute  $\eta = 1$ . The best responses are such that the equations do not have a solution. Therefore, the paper focuses on cases when there is imperfect substitutability in actions.

while the sender anticipates this and acts accordingly. Informationally, this is the worst equilibrium possible amongst the set of all possible equilibria. In this paper, I try to identify conditions under which more informative equilibria emerge.

### 2.3.1 Full Revelation

In a full revelation equilibrium, the private information of the sender is completely revealed to the receiver, meaning  $\mu(\theta) = \theta$  for all  $\theta \in [0, 1]$ . To see if a full revelation equilibrium<sup>10</sup> exists, it is important to understand the trade-offs for the sender. Since actions are substitutable, players compensate for each others action by contributing the residual action required. The players are constrained by the lower and upper bound on permissible actions, given by  $\inf V$  and  $\sup V$  respectively. That is, the size of the bound directly affects the ability of either player to contribute.

Revealing information truthfully then becomes a question of whether the sender can follow up a truthful message with an appropriate action that is within the available domain of the actions. If the sender's action upon truthful communication is within the bound, then it precludes her incentive to misrepresent. Therefore, in some sense, the action set of  $S$  acts as an incentive compatibility constraint for truth-telling.

However, since  $b > 0$  and  $U_{13} > 0$ , it must be that for all  $\theta$ ,  $\phi^S(.) > \phi^R(.)$ . Further, given the assumption of imperfect substitutability ( $\phi_1^{S'}(.) > \phi_2^{S'}(.)$ ), the sender's only concern is whether the optimal best response is within the upper bound of the action set. By single crossing property  $U_{12} > 0$ , therefore, the only incentive compatibility constraint of interest is the one where  $\theta = 1$ . That is, if the optimal action for the sender, upon revealing the highest state, is within the domain of available actions, then it must be so for every  $\theta < 1$ . This property is made clear in the following definition.

---

<sup>10</sup>Any messaging function  $\mu : [0, 1] \rightarrow [0, 1]$  that is one-to-one and onto is a fully revealing messaging strategy. I will, however, concentrate on the most intuitive one in which if the state is  $\theta$ , the sender sends a message that is equivalent to the statement - "The state is  $\theta$ ".

**Definition 2.** Let  $\tilde{x}_S(\theta, m)$  be the optimal actions of the sender when, i) unrestricted domain is satisfied ( $x_S \in \mathbb{R}$ ); and ii) the sender's message  $m$  (truthful or otherwise) is believed by the receiver to be the true state. That is, the action  $\tilde{x}_S(\theta, m)$  is the solution to the unconstrained optimization problem of the sender when her message is believed. Stated formally:

$$\begin{aligned} R's \text{ action: } \tilde{x}_R(m) \text{ solves } \max_{x_R \in V} U(\phi^R(x_R, \tilde{x}_S(m)), m) \text{ subject to } \tilde{x}_S(m) \\ \equiv \arg \max_{x_S \in \mathbb{R}} U(\phi^S(x_S, \tilde{x}_R(m)), m, b) \end{aligned}$$

$$\begin{aligned} S's \text{ action: } \tilde{x}_S(\theta, m) \text{ solves } \max_{x_S \in \mathbb{R}} U(\phi^S(x_S, \tilde{x}_R(m)), \theta, b) \text{ subject to } \tilde{x}_R(m) \\ \equiv \arg \max_{x_R \in V} U(\phi^R(x_R, \tilde{x}_S(m)), m) \end{aligned}$$

Further, when communication is truthful, let the optimal action of players under the unconstrained optimization problem be  $\tilde{x}_R(\theta)$  and  $\tilde{x}_S(\theta) = \tilde{x}_S(\theta, \theta)$ .

**Assumption 4 :**  $\tilde{x}_S(0) \geq \inf V$

Note that Definition 1 does not necessarily prescribe the action of the sender in equilibrium,  $x_S^*(.)$ . Instead,  $\tilde{x}_S(\theta, m)$  allows us to intuitively characterize the optimal response of the sender when her message is believed to be true<sup>11</sup>, and her actions have an unrestricted domain  $\mathbb{R}$ .  $\tilde{x}_S(\theta, m)$  takes into account the fact that the receiver's action<sup>12</sup> is constrained by the bounds imposed by  $V$ . Assumption 4 ensures that the optimal action of the sender is above the lower bound of permissible actions, when she reveals the lowest state. Given the above formulation, let  $c = \sup V$ , be the upper bound of the domain of action set  $V$ . The following conditions are then useful to characterize the communication breakdown and full information revelation equilibrium.

---

<sup>11</sup>One way to interpret this is to think of a naive receiver, as studied in Kartik et al [46] and Ottaviani and Squintani [60]. A naive receiver is not rational, and believes any message sent by the sender.

<sup>12</sup>Since this paper is interested in one-sided incomplete information, analyzing the incentives of  $S$  is easier with this exposition.

**Definition 3.** *Lowest type incentive to separate (LTIS) :  $\tilde{x}_S(0) \leq c$*

**Definition 4.** *Highest type incentive to separate (HTIS) :  $\tilde{x}_S(1) \leq c$*

*LTIS* provides an intuitive criteria for any information transmission with action substitutability. When *LTIS* fails, no information can be credibly revealed by the sender, since the receiver always believes that the sender is exaggerating her information. To put it differently, the sender would always find it profitable to inflate her message so that the receiver contributes more. Instead, if the sender does reveal truthfully, then the actions from the constrained optimization problem (given by equations B.1.1 and B.1.2 in Appendix A.1) and unconstrained optimization problem are such that  $x_S^*(\theta) = c \leq \tilde{x}_S(\theta)$ . The sender, therefore, does not maximize her payoffs from truthful revelation for any  $\theta \in [0, 1]$ .

On the contrary, the *HTIS* condition provides a sort of *IC* constraint for full revelation. As long as *HTIS* is satisfied, the sender can never do better than revealing the truth. This is so because the solution to her constrained optimization problem coincides with that of the unconstrained optimization problem, implying that  $x_S^*(\theta) = \tilde{x}_S(\theta)$  for all  $\theta \in [0, 1]$ . This ensures that there is no incentive for *S* to lie, and full revelation is achieved as an equilibrium.

**Proposition 9.** *Under Assumptions (1-3 and 4), given a bias  $\mathbf{b}$  of the sender,*

1. *No information is credibly revealed in equilibrium if the LTIS condition is violated.*
2. *There is full information revelation if the HTIS condition is satisfied.*

*Proof.* See Appendix B.2 □

Notice that the condition for truth-telling with one-sided incomplete information and strategic substitutability in actions resembles the credibility notion of *self-signaling*<sup>13</sup>, identified by Aumann [9], and Farrell and Rabin [29]. When the unconstrained action is above the bound under truthful revelation, it implies

---

<sup>13</sup>A cheap talk message is self-signaling if the sender intends it to be believed only if it is true.

that the sender faces a positive spillover from the receiver's action, implying that  $U_1(\phi^{S^*}(c, x_R^*(\theta)), \theta, b) > 0$  when  $\tilde{x}_S(\theta) > c$ . This '*positive spillover effect*' implies that communication ceases to be credible at the bound, for the sender (weakly) prefers to induce a higher action from the receiver, by inflating her private information. Baliga and Morris [14] study a game of strategic complementarities in actions, in which the presence of positive spillovers precludes cheap talk communication. In this sense, the first part of Proposition 1 illustrates how the communication breakdown result holds true even when actions are strategic substitutes.

## 2.4 Partial Revelation

So far, I have identified the sufficient conditions for both extremes -communication breakdown and fully revealing equilibria- to emerge. Consequently, as long as *LTIS* holds, there is always some information transmission. The extent of communication therefore depends on how large the upper bound on sender's action is. Proposition 1 implies that when the more stronger *HTIS* condition is satisfied, there can always be a truthful equilibrium. When *HTIS* condition fails, then the sender may only be able to reveal information up to a cutoff, and not beyond. This section will focus on the nature of such '*partial revelation equilibria*' (henceforth PRE)<sup>14</sup>.

Under a PRE, the sender sends a truthful message  $m = \mu(\theta) = \theta$  upto a threshold cutoff  $\theta^*$  and for all states above this cutoff, pools her message ( $m = 1$  if  $\theta > \theta^*$ ). Before stating the result on PRE, one needs to identify the states for which truthful messages can never be credible. As pointed out earlier, these are states for which the IC constraint, given by the upper bound, is violated. Let  $G = \{\theta : \tilde{x}_S(\theta) > c\}$  be the set of states for which truthful revelation is not possible for the sender. The following lemma establishes the condition for  $G$  to be non-empty.

**Lemma 7.** *If *LTIS* holds and *HTIS* is violated, then the set  $G$  is non-empty.*

---

<sup>14</sup>See Appendix A.1 for a more detailed and formal definition.

*Proof.* When  $\tilde{x}_S(0) \geq \inf V$  and  $\tilde{x}_S(1) > c$ , by continuity of  $U(\cdot)$  and  $\phi^i(\cdot)$ , and single-crossing property  $U_{12}(\cdot) > 0$ , the set  $G$  must be non-empty.  $\square$

Given this property, observe that there must then exist a cutoff  $\bar{\theta}$  such that  $\tilde{x}_S(\bar{\theta}) = c$  and  $\bar{\theta} = \sup\{[0, 1] \setminus G\}$ . The set  $G = (\bar{\theta}, 1]$  represents the signal types for which there are incentives to lie for the sender. This is because for all signals in the set  $G$ , reporting the truth implies that the sender's optimal action is outside the domain of permissible actions. Therefore, by misreporting her private information  $\theta$ , say by reporting  $m > \theta$ , the sender can induce the receiver to take a higher action. As a result, none of the messages in this interval are credible and will never be believed in equilibrium. That is, the sender must pool her messages for all signals in  $G$  by sending the highest message<sup>15</sup>,  $m = 1$ . I will now state the result formally.

**Proposition 10.** *Under assumptions 1-4, if LTIS holds and HTIS is violated, then there exists a PRE with threshold  $\theta^* = \bar{\theta}$ .*

*Proof.* See Appendix B.2  $\square$

The PRE expressed above is similar to the cut-off equilibria obtained by Kartik [?], and Morgan and Stocken [?]. In Kartik's work, however, the sender (almost) always uses inflated communication even though the rational receiver is able to invert, and thereby decode, the inflated message. The exaggeration in communication is driven by lying costs. In my result above, the inflated messaging occurs only above the cutoff while every message within the cutoff is truthful. Moreover, the incentive to exaggerate above the cutoff is exacerbated by the fact that any inflated message could now possibly come from a continuum of types, instead of a one-to-one mapping (see proof of Proposition 2 in Appendix). Specifically, any message above the cutoff that is not  $m = 1$  is inverted as being from a set of types, rather than a singleton type, as is the case in Kartik [45]. This ensures that the sender sends

---

<sup>15</sup> $R$  assigns the off-equilibrium path beliefs that would discourage deviations on-the-equilibrium path. For example, for any off-equilibrium path message  $m \in (\bar{\theta}, 1)$ ,  $R$  could assign the state to be the cutoff  $\theta^*$ .



the highest possible message for all types of  $\theta$  above the cutoff, in order to avoid imitation by lower types.

For this reason, notice that above the cutoff  $\bar{\theta}$ , there is no further information communicated. This is in contrast with the result of Ottaviani and Squintani [60], who construct a cutoff equilibrium in which messages are revealing (albeit inflated) below the threshold, and for states above the cutoff information transmission is partitional. In their paper, this is enabled by the presence of naive receivers who interpret messages as they are, implying that even above the cutoff some noisy communication is possible. My PRE result looks similar to the one in their work when the proportion of naive receivers is very low ( $< \frac{1}{4}$  in their characterization), in which case there are no partitions above the cutoff and messages are fully revealing below the threshold.

Introducing naive receivers in my setup does not change the nature of equilibria, in the sense that partitions above the cutoff, similar to the ones constructed in Ottaviani and Squintani [60], can never be achieved. However, note that adding naive receivers increases the threshold. To see this, say the receiver is naive (with probability 1). Then, because  $\tilde{x}_S(\bar{\theta}, 1) < c$  and  $\tilde{x}_S(1, 1) > c$ , by continuity property of  $U(\cdot)$  and single crossing, there must be a threshold  $\tilde{\theta} > \bar{\theta}$  such that  $\tilde{x}_S(\tilde{\theta}, 1) = c$ . Therefore, adding naive receivers to my setup increases the most informational threshold of PRE.

#### 2.4.1 Multiplicity of threshold equilibria

In cheap talk models, since messages are costless and do not affect the utility function directly, there always exists a babbling equilibrium in which no information is communicated. The analysis so far points out that when the LTIS condition fails, then the only equilibrium of the game is a babbling one (the case of communication breakdown). When LTIS condition holds but the HTIS instead is violated, there is always a PRE of the form described above. This cutoff equilibrium  $\theta^* = \bar{\theta}$ , however,

is only one of many possible PRE that could emerge. To characterize all the PRE and simplify analysis of the same, I will make the following assumption:

**Assumption 5 :**     *Feasible Low type deviation:*  $\tilde{x}_S(0, 1) \in V$

This assumption ensures that when the lowest type sender ( $\theta = 0$ ) misrepresents her signal and sends the most inflated message ( $m = 1$ ) that is further believed to be true by the receiver, the optimal (unconstrained) action of the sender is within the domain of permissible actions. Note that this is a stronger version of assumption 4, which concerns only with the feasibility of truthful communication of the lowest type information. The reason why Assumption 5 is useful is that it then allows us to characterize the PRE with  $\theta^* < \bar{\theta}$  in an intuitive way. The assumption makes upward deviations by sender possible in the sense that they would never induce an action that goes below the lower bound of the feasible action set, meaning  $\tilde{x}_S(0, m) \in V$  for all  $m < 1$ . Given this, the following proposition characterizes all the cutoff PRE of the game.

**Proposition 11.** *Under Assumptions 1-5, if LTIS holds and HTIS is violated, then every threshold  $\theta^* \in (0, \bar{\theta})$  is a PRE.*

*Proof.* See Appendix B.2 □

One implication of the above result is that the sender could possibly choose how much information to reveal in equilibrium. Though the PRE  $\theta^* = \bar{\theta}$  is the most informational one, it does not necessarily restrict the sender from providing lesser information. In section 5, I address this multiplicity problem pertaining to PRE's, by looking at the welfare properties of the different threshold equilibria. In the following section, I attempt to provide some comparative statics results that throw light on the most informative PRE,  $\theta^* = \bar{\theta}$ .

## 2.5 Comparative Statics

As pointed above, the PRE relies on two crucial parameters - the bias of the sender and the size of the upper bound  $c$ . In both cases, what is relevant is to check how the most informative PRE reacts to changes in these parameters. With bias  $b$ , the results from the standard literature on cheap talk holds in my setting as well. This is fairly straightforward and not surprising to notice. Specifically, if  $b$  decreases, then since  $U_{13} > 0$ , it implies that the sender is able to compensate sufficiently for more types in the state space. That is, when  $b_1 < b_2$ , then the most informative equilibria under the two biases, respectively, would be such that  $\bar{\theta}_1 > \bar{\theta}_2$ .

The more interesting comparative static finding comes from varying the extent of the bound  $c$ , or in other words, expand the domain of actions imposed on the sender. Remember that  $c$  affects truth telling by enabling the sender to compensate for types up to a certain threshold. Increasing this bound leads to more communication since the sender can now reveal truthful information for more types, pushing the threshold to the right. I summarize these two intuitive findings in the following claim.

**Claim 1.** *Take two biases  $b_1$  and  $b_2$  such that  $b_1 < b_2$ . Then the most informative PRE under the two biases are such that  $\bar{\theta}_1 > \bar{\theta}_2$ .*

**Claim 2.** *Take two bounds  $c_1 = \sup V_1$  and  $c_2 = \sup V_2$  such that  $c_1 < c_2$ . Then the most informative PRE under the two bounds are such that  $\bar{\theta}_1 < \bar{\theta}_2$ .*

The bounds could be interpreted as a form of resource or capacity constraint on the players. In the context of the examples put forth in the introduction, we could think of a lower  $c$  as a type of (capacity) cost imposed on the sender that prevents her from revealing information. Propositions 1 and 2, therefore, highlight the importance of the interaction between resource constraints and the bias of the sender when there is action substitutability.

Resource constraints are frequently observed in the real world. As a motivation, think of the following scenario. Suppose two departments in an organization decide to implement a project that involves contributions from both entities. Further, let us assume that only one department holds information relevant to the implementation of the joint project, and contributions are substitutable. In this situation, an absence of any constraints (shortage of manpower or financial burdens, say) would enable the two departments to perfectly cooperate its activities, and all private information regarding the project may be credibly conveyed. However, in the presence of resource constraints, the informed department could misrepresent its information for higher states of the world, in order to induce the other to spend more resources on the project. Any information loss could then be viewed as a source of inefficiency in the project. Above results show that in order to improve informational efficiency, it may be in the interest of an informed party to either choose a partner with more aligned interests, or mitigate the burden of constraints imposed upon it by the organization<sup>16</sup>.

## 2.6 Welfare

The previous section establishes that more information can be revealed when the upper bound of the domain of actions available to the sender is increased. However, the sender may also choose to reveal any threshold of information, starting from a cutoff  $\theta^* = 0$ , up to a  $\theta^* = \bar{\theta}$ . An important question that arises is would the sender find it in her interest to convey less information. Given a cutoff  $\theta^*$ , the ex ante utility of receiver  $R$  can be expressed as,

---

<sup>16</sup>In a typical organization, this could involve hiring more staff for the project or increasing the budget allocated for the project within the department.

$$V_R(\theta^*) = \int_0^{\theta^*} U(\phi^{R^*}(x_R^*(t), x_S^*(t)), t) f(t) dt + \int_{\theta^*}^1 U(\phi^{R^*}(x_R^*((\theta^*, 1]), x_S^*(t, (\theta^*, 1])), t) f(t) dt$$

That of sender  $S$  can be similarly written as,

$$V_S(\theta^*) = \int_0^{\theta^*} U(\phi^{S^*}(x_S^*(t), x_R^*(t)), t, b) f(t) dt + \int_{\theta^*}^1 U(\phi^{S^*}(x_S^*(t, (\theta^*, 1]), x_R^*((\theta^*, 1])), t, b) f(t) dt$$

where,  $x_S^*(t, (\theta^*, 1])$  and  $x_R^*((\theta^*, 1])$  are the equilibrium actions of the sender and receiver respectively, given the receiver's beliefs that the state is in the interval  $(\theta^*, 1]$ , given a cutoff  $\theta^*$ .

Notice that a higher  $\theta^*$  may benefit the receiver since providing more accurate information over a larger domain of type space makes it possible for the receiver to compensate more precisely for these (truthfully) reported states. But does this hold from the perspective of the sender? Remember that the sender initially holds an informational advantage. This is further exacerbated by the fact that she can always undo the actions of the receiver even after the information is revealed. Providing more information to the receiver need not therefore affect her own payoffs, as long as the action set available to her allows contributing the best response action. This ability to take an action after the communication changes her incentives to reveal more information during the communication stage. In fact, these observations provide for an intuitive ranking of the PRE's, as stated in the following proposition.

**Proposition 12.** *The most informational PRE where  $\theta^* = \bar{\theta}$  Pareto dominates*

every other PRE.

*Proof.* See Appendix B.2 □

The intuition for this Pareto ranking of equilibria is seen by extending the arguments made above. Specifically, think of a sender providing information upto some threshold, say  $\varepsilon$ . Then, for all other states  $\theta > \varepsilon$ , the sender, by pooling her message on  $m = 1$ , induces an expectation over the possible states for the receiver given by beliefs that  $\theta \in (\varepsilon, 1]$ . The action of the receiver takes into account this posterior belief and induces an action  $x_R^*((\varepsilon, 1])$ . Notice that for any such cutoff  $\varepsilon$ , there must exist a  $\theta_\varepsilon$  such that  $x_S^*(\theta_\varepsilon, (\varepsilon, 1]) = c$ , by assumption 5 and single crossing. But, for every  $\theta > \theta_\varepsilon$ , the sender suffers inefficiency since she is unable to compensate sufficiently. Pareto ranking the equilibria then becomes possible by observing two sources of inefficiency that arises with pooling of information. First,  $\theta_\varepsilon$  is decreasing in the amount of information pooling. That is, the greater the cutoff  $\varepsilon$ , the smaller are the measure of types  $(\theta_\varepsilon, 1]$  for which the sender is unable to compensate efficiently. Second, the severity of this inefficiency for states  $\theta > \theta_\varepsilon$ , given that  $x_S^*(\theta, (\varepsilon, 1]) = c$ , is greater when  $\varepsilon$  is smaller. Both these sources of inefficiencies are reduced when more information is communicated in equilibrium. As a result, it is always in the sender's interest to provide more information in equilibrium.

Usually, however, ex ante Pareto dominance is not a sufficient criterion to select equilibria since it only provides a aggregate welfare measure. In particular, it may be that different types of sender may have varying preferences over the equilibria, making it harder to tackle the multiplicity problem prevalent in cheap-talk models<sup>17</sup>. However, when actions are strategic substitutes, I find that a higher cutoff PRE is not only ex-ante efficient, but also guarantees interim efficiency. That is, every sender type weakly prefers a higher cutoff PRE to a lower one. This,

---

<sup>17</sup>In view of this, Chen, Kartik and Sobel [23] develop a selection criterion to deal with multiplicity problem in the classic CS setup - a notion they call the '*No incentive to separate*' condition.

combined with Proposition 4, implies that not only is the aggregate expected utility higher under  $\bar{\theta}$  equilibrium, but it is also the case that every type of sender is (ex ante) weakly better off under  $\bar{\theta}$ .

**Proposition 13.** *Every sender type (ex ante) weakly prefers a PRE with  $\theta^* = \bar{\theta}$ .*

*Proof.* See Appendix B.2 □

The intuition is an extension of the arguments made for Proposition 4. Specifically, sender types that are at the higher end of the spectrum tend to prefer  $\bar{\theta}$  since there is a positive spillover effect at the bound. Hence, for these high types, inducing a higher (expected) action from the receiver increases payoffs. Since the most informational equilibrium  $\bar{\theta}$  induces a greater action from the receiver, meaning  $x_R^*((\bar{\theta}, 1]) > x_R^*((\theta^*, 1])$  for all  $\theta^* < \bar{\theta}$ , the highest types strictly prefer the PRE with cutoff  $\bar{\theta}$ . For all other types the optimal response is within the bound, making them indifferent between  $\bar{\theta}$  and  $\theta^*(< \bar{\theta})$ . Therefore, either every type of sender is indifferent to, or strictly prefers a PRE with  $\bar{\theta}$ .

## 2.7 Conclusion

This paper investigates the nature of cheap talk communication with (one sided) incomplete information when actions of players are strategic substitutes. With pure messaging strategies, I show that cheap talk communication fully reveals information when the informed sender is able to compensate for the actions of the uninformed receiver for every possible state, once that private information is revealed. Conversely, when the sender is unable to reveal even the lowest state truthfully, there is complete communication breakdown and no information is conveyed.

Consequently, when the domain of action set constrains the sender from taking an efficient action for some states, I find that communication deteriorates. Specifically, there is only partial information revelation in equilibrium. Full revelation breaks down, since the sender now has an incentive to exaggerate her private

information for higher realizations of the state. This results in a communication equilibrium in which there is truthful revelation up to a threshold, and for states higher than this threshold, no information is conveyed. An interesting property of the equilibria is that they are dependent on the boundedness of action sets. Specifically, the bounds on actions act as an incentive constraint for truthful revelation.

The framework I have presented could be extended to applications with multiple senders and (or) receivers (contribution games in groups, team theory, for example). With multiple sources of information, the nature of truth-telling would be dependent on the distribution of the biases and the size of the constraints on actions. In case of multiple receivers, the sender's incentive to reveal information would be similar to the conditions I developed in this paper. This means that full revelation ensues as long as the highest type signal can be credibly revealed by the sender. Such scenarios require a more detailed analysis, and is left for future work.

Another interesting avenue to explore is the role of commitment on part of the receiver (see, e.g., Alonso and Matoushek [6], and Melumad and Shibano [51]). Can the receiver, by committing to a message contingent decision rule (a *deterministic mechanism*), do better than the cheap talk equilibrium? Of course, when there is full information revelation, the receiver can do no better from commitment. However, in the case of partial revelation, whether a mechanism of the kind described above is better for the receiver remains an open question for future research.



## Chapter 3

# An Informational Theory of Alliance Formation

### 3.1 Introduction

Countries form alliances with each other in order to achieve common goals (e.g., military, security, and economic). Examples of such modern day alliances –military and otherwise– include NATO, The EU, ASEAN, among others. These alliances are cooperative in nature in that they consist of countries that have an overarching set of policy goals, who then act cooperatively in realizing those shared objectives<sup>1</sup>. At the heart of such cooperation, then, is the informational incentive of being part of an alliance<sup>2</sup>. Sharing information, be it external intelligence, or internal security related, is therefore a vital component of alliances between sovereign nations. Specifically, this captures an environment in which individual member nations strategically share

---

<sup>1</sup>NATO's 2010 Strategic Concept [25] document specifies this idea succinctly, and we quote - "The Alliance will engage actively to enhance international security, through partnership with relevant countries and other international organisations; by contributing actively to arms control, non-proliferation and disarmament."; further, it adds "Any security issue of interest to any Ally can be brought to the NATO table, to share information, exchange views and, where appropriate, forge common approaches."

<sup>2</sup>Traditionally, in the international relations literature (Walt [69] [70], and Waltz [71]), alliance formation has primarily been studied within the purview of state capacity - either align in order to balance against a powerful state or bandwagon with a threatening state (or coalition).

information through diplomatic channels, and take appropriate actions that is commensurate with the information aggregated and the preferences of other members.

We develop a model of alliances that incorporates four key features: information sharing, strategic interdependency in actions, preference heterogeneity, and resource constraints. Initially, members of an alliance receive a private signal about an unknown state of the world that affects their payoff. In the communication stage, each player, publicly and simultaneously, sends a cheap-talk message about their private information to the group. After the communication stage, conditional on the private information and the messages exchanged, each player takes an action, where actions of players are strategic substitutes.

Our main finding is the full information aggregation result. Specifically, we show that all private information held by members of an alliance are revealed in equilibrium as long as players' biases are cohesive – the distance between the bias of an individual player and a weighted average of biases of the group falls within a certain bound. This result generalizes the earlier results from Chapter 2 for the case of  $N > 2$  players. The intuition behind the result is that as long as players' biases are cohesive, each player cannot do better than fully revealing her private information. This way, as long as the players' available domain of actions is large enough, the private information held by members of the alliance is fully aggregated.

The paper closest to our work is the one by Galeotti et al. [33]. Though the information and communication structure are identical, a fundamental difference is that in their work, actions of players were independent of each other. As a result, the message of a player does not affect her own actions. In our setup, since actions are interdependent, a player's message also affects her beliefs about other players' actions, and therefore, affects her own action. Hagenbach and Koessler [39] study a model of strategic with multiple players and interdependent actions. However, two differences emerge. First, while they study strategic complementarities in actions, we develop a model in which actions are substitutable. Second, in their information

framework, private signals of players are independent and communication is private. On the other hand, we are interested in a model where signals are correlated, but the communication protocol is public. As a result, our analysis is very distinct from either of the two papers mentioned above.

An important theoretical contribution of our work is the fact that the domain of the action set of players –resource constraint– drives truthful communication. In particular, when actions are unrestricted (no resource constraints), there is always truthful communication irrespective of the bias differences. In this sense, we find a novel interaction between bias dispersion and resource constraints, which was absent in both Galeotti et al. [33] and Hagenbach and Koessler [39].

The rest of the paper is organized as follows. Section 3.2 develops the framework of the model and section 3.3 presents the full information aggregation result. Section 3.4 provides brief concluding remarks.

## 3.2 Model

A group of players,  $N = \{1, 2, \dots, n\}$ , decide on contributions to a joint project. Each player chooses an action  $x_i \in [0, 1]$ , where the bounds represent the resource constraint faced by every individual player. Moreover, actions of players are themselves interdependent in a way that they are strategic substitutes. The payoff of every player is dependent on an unknown common state of the world  $\theta$  distributed uniformly on  $[0, 1]$ . The state  $\theta$  is not directly observable, but each player  $i$  receives a private signal  $s_i \in \{0, 1\}$  about the state of the world such that:  $s_i = 1$  with probability  $\theta$ , and  $s_i = 0$  with probability  $1 - \theta$ . Finally, each player has a bias  $b_i$ , that captures the extent to which a player cares about the outcome (without loss of generality,  $0 \leq b_1 \leq b_2 \leq \dots \leq b_n$ ).

Formally, player  $i$ 's utility is given by,

$$u_i(\mathbf{x}; \theta, b_i) = -\left[\left(\frac{x_i + \eta x_{-i}}{1 + (n-1)\eta} - \theta - b_i\right)^2\right]; \text{ where } x_{-i} = \sum_{j \neq i} x_j, \mathbf{x} = (x_1, x_2, \dots, x_n)$$

This utility form captures the four features described earlier. First, players' actions are interdependent in that utility depends on deviations of the (player-specific) joint contribution function from the player's ideal action, given by  $\theta + b_i$ . This interdependence is such that  $\frac{\partial^2 u_i(\cdot)}{\partial x_i \partial x_j} < 0$ , and the degree of substitutability is not perfect, ie,  $\eta \in (0, 1)$ . Second, there is a need for information sharing in order to aggregate each players' private information about the state  $\theta$ . Third, players face a resource constraint since there are signal realizations and biases such that  $\theta + b_i > 1$ , but actions are bounded on  $[0, 1]$ . Lastly,  $b_i$  captures the preference heterogeneity of players over final outcomes<sup>3</sup>.

This set-up lends itself naturally to situations in international affairs that involve countries cooperating with each other to resolve a common foreign policy objective - like engaging in conflicts, or providing assistance to peacekeeping, among others. In such scenarios, each country in the alliance has potentially varying degrees of information and interest in the cause. Sharing private information enables countries in an alliance to target resources in an efficient way, and doing so is a vital component of successful cooperation<sup>4</sup>.

### 3.2.1 Communication Round

With no communication, player types are correlated and there is complementarity in players' signals<sup>5</sup>. Each player, before the communication stage, can be classified into one of two types - high ( $s_i = 1$ ) and low ( $s_i = 0$ ). We allow for communication in the following way: after each player receives their signal  $s_i$ , they publicly and

<sup>3</sup>These features are perfectly encapsulated by the NATO Strategic Concept 2010 document. Specifically, it identifies three key objectives of the security alliance between NATO countries - Collective Defence, Crisis Management and Cooperative Security. The emphasis is on 'cooperation' and 'collective', and achieving this requires a way to deal with preference heterogeneity within the alliance and structure to communicate information among the members.

<sup>4</sup>For example, consider NATO's Partnership Action Plan against Terrorism, drafted post the September 2011 attacks. It clearly delineates the vital element of information sharing as one of the key requirements for effectively fighting terrorism and other security related challenges. For more, see [http://www.nato.int/cps/en/natohq/official\\_texts\\_19549.htm](http://www.nato.int/cps/en/natohq/official_texts_19549.htm).

<sup>5</sup>In our model, player signals are conditionally independent. However, there is signal correlation and complementarity in the following way:  $\Pr(s_j = 1 \mid s_i = 1) = \frac{2}{3}$ ,  $\Pr(s_j = 0 \mid s_i = 1) = \frac{1}{3}$  and  $\Pr(s_j = 0 \mid s_i = 0) = \frac{2}{3}$ ,  $\Pr(s_j = 1 \mid s_i = 0) = \frac{1}{3}$ .

simultaneously communicate their information through cheap-talk messages  $m_i \in M_i$  to each of the other  $n - 1$  players.

In this paper, we focus on pure messaging strategies and a public communication protocol, in which each player simultaneously sends a public message  $m_i(s_i)$  to every other player in the group<sup>6</sup>. Player  $i$ 's messaging strategy is given by,

$$m_i : \{0, 1\} \longrightarrow \{0, 1\}$$

A truthful message by  $i$  to the group implies  $m_i(s_i) = s_i$  for  $s_i = 0$  and  $1$ , and babbling message is one where  $m_i(s_i) = m_i(1 - s_i)$ . Let  $\mathbf{m} = (m_1, m_2, \dots, m_n)$  be the communication strategy of the  $n$  players<sup>7</sup>. Through out this paper, we abstract away from other more complicated forms of messaging strategies<sup>8</sup>, and focus on pure communication strategies for reasons of tractability and clearer exposition of the trade-off's.

### 3.2.2 Action round

Once messages have been exchanged, each player decides on their individual contribution  $x_i$ . We will once again focus on pure second-stage strategies. Since the utility function is strictly concave in  $x_i$ , best-responses exist and are unique. A second stage strategy can be defined as follows:

$$\tau_i : S_i \times (M_i \times M_{-i}) \rightarrow [0, 1]$$

Therefore,  $\tau_i(s_i, (m_i, m_{-i}))$  is the strategy of player  $i$  with signal  $s_i$ , having sent message  $m_i$  and received messages  $m_{-i} = (m_j)_{j \in N \setminus \{i\}}$  from the group. Let  $\tau(\mathbf{s}, \mathbf{m}) = (\tau_i(s_i, (m_i, m_{-i})))_{i \in N}$  be the strategy profile of the players.

---

<sup>6</sup>Public communication protocols are very common in the real-world. For example, forums like UN, NATO and other regional alliances often get together and share private information about a common issue. Public diplomacy remains a main feature of such organizations.

<sup>7</sup>Both the signaling structure and messaging strategies are similar to Galeotti et al. [33] In fact, to be more precise, they use a more general communication protocol, placing no restriction on messages being public or private.

<sup>8</sup>For example, one such mixed strategy would be a partially separating strategy under which player  $i$  babbles (or reveals) for one signal type and mixes between truth-telling and babbling for the other signal type.

### 3.2.3 Equilibrium Definition

Given the above structure of messaging, the players can be grouped post the communication round into two sets (according to equilibrium beliefs) - truthful set and babbling set. We define them in the following way:

**Definition 5.** *Truthful set*,  $T = \{i : m_i(0) = 0, m_i(1) = 1\}$

**Definition 6.** *Babbling set*,  $B = \{j : m_j(0) = m_j(1)\}$

The first is just the set of players whose messages are believed in equilibrium as informative, and messages from the second are ignored as uninformative (note that all this is based on equilibrium beliefs). Given this, the vector of messages after communication consists of  $|T|$  truthful messages  $m_T = \{m_i : i \in T\}$  and  $|B|$  babbling messages  $m_B = \{m_j : j \in B\}$ . Note that any off-equilibrium path messages are believed and treated as if they were equilibrium messages. This gives rise to an *IC* constraint for truth-telling such that, in equilibrium, each player's beliefs about other players' messages are updated using Bayes' rule.

The equilibrium concept is sequential equilibrium in pure strategies. An equilibrium is defined as a strategy profile  $(m, \tau) = ((m_i)_{i \in N}, (\tau_i)_{i \in N})$  such that,

1. Actions are sequentially rational, given messages and beliefs:

$$\forall i \in N, m_{-i} \in M_{-i} :$$

$$\tau_i(s_i, (m_i, m_{-i})) \in \arg \max_{x_i} \int_0^1 \sum_{s_{-i} \in \{0,1\}^{n-1}} u_i(x_i, (\tau_j(s_j, (m_j, m_{-j})))_{j \neq i}; \theta, b_i) \Pr(s_{-i} \mid \theta) f(\theta \mid m_{-i}, s_i) d\theta$$

2. Messages are truthful iff they satisfy the *IC* for truth-telling:

$\forall i \in N, s_i \in \{0, 1\} :$

$$\begin{aligned}
& - \int_0^1 \sum_{s_{T-1} \in \{0,1\}^{t-1}} \sum_{s_B \in \{0,1\}^{n-t}} u_i(\tau_i(s_i, (s_i, m_{-i})), (\tau_j(s_j, (s_i, m_{-i})))_{j \in T-1}, \\
& \quad (\tau_k(s_B(k), (s_i, m_{-i})))_{k \in B}; \theta, b_i) f(\theta, s_{T-1}, s_B | s) d\theta \\
& \geq \\
& - \int_0^1 \sum_{s_{T-1} \in \{0,1\}^{t-1}} \sum_{s_B \in \{0,1\}^{n-t}} u_i(\tau_i(s_i, (1 - s_i, m_{-i})), (\tau_j(s_j, (1 - s_i, m_{-i})))_{j \in T-1}, \\
& \quad (\tau_k(s_B(k), (1 - s_i, m_{-i})))_{k \in B}; \theta, b_i) f(\theta, s_{T-1}, s_B | s) d\theta
\end{aligned}$$

where  $s_{T-1}$  is the set of  $(T-1)$  truthful signals, apart from player  $i$  and  $s_B$  is the set of babbling signals

### 3.3 Equilibrium Characterization

We proceed by first characterizing the optimal best responses in the contributions stage of the game. When deciding on how much to contribute given the information generated by communication, each player takes into account the interdependence in actions. Let  $t = |T|$  and  $b = |B|$  be the number of truthful players and babbling players respectively, after the communication stage.

Another intuitive way of thinking about the action stage is to abstract away from communication, and assume the following. Suppose that all agents were exogenously given information  $m_T$ , and a sub-group of  $b$  agents were additionally provided with a private signal - 0 or 1. Given this exogenous information structure, what are the optimal actions of each player given their information, and the interdependence in actions. The solution to the problem is then a Bayesian Nash equilibrium (BNE) in which there are  $b$  players who can be either of two types, and  $t$  truthful players.

The maximization problem of each of the  $t$  truthful players is given by,

$$\forall i \in T : \max_{x_i} E_{\theta, s_B} [u_i((x_i, x_{T \setminus \{i\}}, x_B(s_B)); \theta, b_i) \mid m_T] \quad (3.3.1)$$

where,  $x_{T \setminus \{i\}} = \{x_j : j \in T \setminus \{i\}\}$  and  $x_B(s_B) = \{x_j(s_B(j)) : j \in B\}$  are the vector of actions by the  $t - 1$  truthful players other than  $i$ , and  $b$  babbling players respectively.

Analogously, the maximization problem for a babbling player  $j$  with private signal  $s_j$  is given by,

$$\forall j \in B, s_j \in \{0, 1\} : \max_{x_j(s_j)} E_{\theta, s_B} [u_j((x_j(s_j), x_T, x_{B \setminus \{j\}}(s_{B \setminus \{j\}})); \theta, b_j) \mid m_T, s_j] \quad (3.3.2)$$

where,  $s_{B \setminus \{j\}} \in \{0, 1\}^{n-t-1}$  is the vector of all possible signals of the remaining  $(n - t - 1)$  babbling players,  $x_T$  is the vector of actions of all the truthful players, and  $x_{B \setminus \{j\}}(s_{B \setminus \{j\}})$  is the vector of actions of remaining  $(n - t - 1)$  babbling players aside  $j$ . Hence, players choose an action that solves a system of equations  $(t + 2b)$  given by their maximization problem<sup>9</sup>, as stated above.

### 3.3.1 Characterization of equilibrium contributions

#### No Resource constraints

We proceed by first characterizing the best responses of each type of player and their equilibrium actions, as if there were no resource constraints on the actions of players. The reason to do this is twofold. First, it abstracts away from the difficulty of thinking about interdependent actions with bounds, and allows us to find closed-form solutions to the action stage problem. Second, the exact form of equilibrium

---

<sup>9</sup>The usefulness of looking at public communication can be seen from the above equations. Every player in the group knows precisely who the set of truthful players are (in equilibrium beliefs), their messages, as well as the set of babbling players. Moreover, given the beta-binomial distribution, each player can then form expectations of what private information any babbling player holds. The babbling player is then one of two types - low signal or signal type - and every player in the group has the same posterior about the conditional expectation over babbling types.



actions, as will be made clearer later, provides important intuition to think about the messaging strategies of players and characterize the messaging equilibrium.

We also impose a bound on the dispersion of biases so that the alignment of any player  $i$  remains within certain limits<sup>10</sup>. Specifically, we define the following:

**Definition 7.** Let  $A_i = [b_i - \frac{\eta}{(1+(n-1)\eta)} \sum_{j \in N} b_j]$ , be a measure of dispersion of the alignment of interests of player  $i$  from that of the group. Further, assume that  $\forall i \in N, \frac{(1+(n-1)\eta)}{1-\eta} \cdot A_i \in (-1, 1)$ .

**Lemma 8.** Under unrestricted domain ( $x_i \in \mathbb{R}$ ) and public communication, the players' sequentially rational action after receiving  $t$  truthful messages and  $(n-t)$  babbling messages is given by:

*Truthful player:*

$$x_{i \in T} = \frac{(1 + (n-1)\eta)}{1 - \eta} \cdot A_i + \frac{(k+1)}{(t+2)}$$

*Babbling player with low signal:*

$$x_{(i \in B, s_i=0)} = \frac{(1 + (n-1)\eta)}{1 - \eta} \cdot A_i + \frac{k+1}{t+2} \cdot \frac{h(t)}{1+h(t)}$$

*Babbling player with high signal:*

$$x_{(i \in B, s_i=1)} = \frac{(1 + (n-1)\eta)}{1 - \eta} \cdot A_i + \frac{k+1}{t+2} \cdot \frac{h(t)}{1+h(t)} + \frac{1}{1+h(t)}$$

$$\text{where } h(t) = \frac{\frac{(2+t(1-\eta))}{(1+(n-1)\eta)}}{\left(1 + \frac{(2+t(1-\eta))}{(1+(n-1)\eta)}\right)}$$

Notice that the actions post communication is dependent on  $A_i$ , the difference in bias of the player from the weighted average of biases of all players in the group.

<sup>10</sup>Since we consider communication with resource constraints, it is necessary to limit the values a player's bias can take. For example, we cannot have one player to have an extremely high bias, like  $b_i = 10$  or so. This trivializes the problem when bounds are introduced, since, irrespective of the communication, player  $i$  always takes the maximum action within the bound,  $x_i = 1$ . Moreover, every such player with extreme biases always misreport their signal in equilibrium.

We construe this difference as a measure of alignment of interests in the group, or alternatively, as a measure of dispersion in the biases.

When players' actions have unrestricted domain (meaning  $x_i \in \mathbb{R}$ ), players are able to exactly compensate for the messages in equilibrium and possibly 'undo' the effects of communication by choosing an optimal action that exactly matches their ideal state, given the set of equilibrium messages. Under unrestricted domain, there always exists a fully revealing equilibrium in which every player reveals her private information to the group<sup>11</sup>.

In a fully revealing equilibrium with unrestricted domain of actions,  $T = N$  and every player plays the following action, post the communication round:

$$x_{i \in N} = \frac{(1 + (n-1)\eta)}{1 - \eta} \cdot A_i + \frac{(k+1)}{(t+2)} \quad (3.3.3)$$

### Resource constraints

Intuitively, introducing resource constraints by restricting the set of actions to  $[0, 1]$  changes the nature of information revelation for the players by reintroducing trade-off's between providing more information and concerns of under (or over) provision. This arises because with bounded actions, players are unable to completely compensate for the effect of their message on the actions of others.

To make this point clearer, let us consider the action of  $n$  - the player with the highest bias. In the fully revealing equilibrium described above, under a restricted action set  $[0, 1]$ ,  $x_n = \min\{1, \frac{(1+(n-1)\eta)}{1-\eta} \cdot A_i + \frac{(k+1)}{(t+2)}\}$ . This implies that  $x_n = 1$  whenever the other expression exceeds the bound. This leads to under-provision as other players would not substitute completely (since  $\eta < 1$ ), leaving player  $n$  to suffer a loss from under-provision. The problem of under-provision is exacerbated

---

<sup>11</sup>This point has been made in Chapter 2. Specifically, it shows that, for a specific class of utility functions, when players' actions are unrestricted, there is completely truthful communication as players can undo the effects of communication in the subsequent action stage. This ability to compensate for the messages ex-post precludes the incentives to lie and ensures truthful communication.

when player  $n$ 's signal is  $s_n = 0$ . Fearing under-provision, player  $n$  can do better by exaggerating her private information and sending a message  $m_n = 1$  to the group instead. This type of exaggeration has two effects. First, there is a pure information effect that pushes every players' actions up. Second, there is a countervailing free-riding effect in that players now also understand that  $n$  is also going to take a higher action, and hence will adjust their actions accordingly. Nevertheless, player  $n$  benefits from misreporting since even in equilibrium, each of the other players' action are higher (in expectations), and this reduces the loss from under-provision. In equilibrium, though, player  $n$ 's message is never credible for precisely the above reasons, and thus will not be believed.

Players with very low biases in the group face the opposite problem - that of over-provision. Without loss of generality, take the case of player 1 with bias  $b_1$ . Again, when each of the other  $(n - 1)$  players are revealing truthfully, say, player 1 may have an incentive to deviate from reporting a high signal ( $s_1 = 1$ ) truthfully. Player 1's optimal action as dictated by the previous lemma is  $x_1 = \max\{0, \frac{(1+(n-1)\eta)}{1-\eta}A_i + \frac{(k+1)}{(t+2)}\}$ . When, however, for any possible signal realization  $s_{N \setminus \{1\}}$ , the optimal action of player 1 is below zero, then the bounds kick in and  $x_1 = 0$ , leading to over-provision concerns. That is, player 1 can benefit from under-reporting her high signal and instead send a message  $m_1 = 0$ . This would push the actions of the rest of the players in group down, thereby decreasing losses from over-provision.

Therefore, with resource constraints, two types of problems arise with communication. Players with a higher preference, in order to avoid under-provision, may tend to exaggerate their private information and those with lower biases, fearing over-provision, may end up under-reporting their signals. One question that arises naturally in this context is who exactly are the players with under (over) provision concerns. A closer look at the equilibrium actions under unrestricted domain provides crucial intuition for answering this question.

For every player  $i$  such that  $A_i < 0$ , actions in any equilibrium can never hit the upper bound. However, there may be truthful signal realizations under which the optimal action may be less than 0, but because of the lower bound on actions, player  $i$  would be constrained to play  $x_i = 0$ . This implies for these players, there are over-provision concerns, meaning they would always report their low signal truthfully, but under-report the high one. Similarly, the players for whom  $A_i > 0$  would worry about the upper bound as their actions are always positive for any set of signal realizations. These players are ones who fear under-provision and therefore, have an incentive to exaggerate their low signals.

Therefore, players themselves can be separated into two types -  $0 - type$  and  $1 - type$ . We define them in the following way:

**Definition 8.**  $0 - type = \{i \in N : A_i < 0\}$

**Definition 9.**  $1 - type = \{i \in N : A_i > 0\}$

The players in the set  $0 - type$  always reveal their low signal, but face incentives to misrepresent  $s_i = 1$ . Vice versa, the players in the set  $1 - type$  always reveal their high signal, but may misrepresent their low signal,  $s_i = 0$ .

### 3.3.2 Full Information aggregation with resource constraints

In the previous subsection, we put forth some of the trade off's involved in information revelation with resource constraints. Particularly, the set of players can be (ex-ante) partitioned into either a  $0 - type$  or  $1 - type$ , depending on the initial distribution of biases  $\mathbf{b}$ . Further, we know that with public communication and pure messaging strategies, information revelation takes the form of a partition of truthful players  $T$  and babbling player  $B$ . The natural question that arises is, under what conditions does there exist full information aggregation with resource constraints. The following Lemma provides the necessary and sufficient conditions required for complete information aggregation ( $|T| = n$ ) under resource constraints:

**Theorem 1.** *Under public communication protocol with given bias distribution  $\mathbf{b}$  and resource constraints, there is a  $n$ -player equilibrium such that every player in the group reveals truthfully if and only if:*

$$\forall i \in N : \quad |A_i| \leq \frac{2}{n+2} \cdot \frac{(1-\eta)}{(1+(n-1)\eta)}$$

*Proof.* See Appendix C □

We provide brief intuition for the above result. To do so, we borrow from earlier arguments as well as from Austen-Smith [12]. For a player in the group to reveal information truthfully, it must be that for every possible signal type, and every possible signal realization of the other  $(n-1)$  players, her action must be within the bounds. For a  $0$ -type player, the relevant one is the lower bound of  $0$ , and for the  $1$ -type, upper bound of  $1$ .

Let a player, say  $i$ , from the set  $0$ -type hold a signal  $s_i = 1$ . In any equilibrium where  $t$  players reveal truthfully, it must hold that the equilibrium action of player  $i$  is greater than zero, for every possible signal realization<sup>12</sup> of the remaining  $(t-1)$  truthful players. Otherwise, there is an incentive to lie for player  $i$  in the communication stage. Moreover, what matters is the sufficient statistic (given by  $\sum_{j \in T \setminus \{i\}} s_j$ ) of the  $(t-1)$  signals. Given this formulation, truth-telling for a  $0$ -type player has to satisfy the tightest  $IC$  constraint - meaning her action under the tightest constraint has to be within the bounds. If this was not so, an equilibrium with  $i \in T$  violates the  $IC$  for truth-telling. The tightest constraint for a  $0$ -type player occurs when  $k = 1$ , meaning all other  $(t-1)$  signals are  $0$  ( $\sum_{j \in T \setminus \{i\}} s_j = 0$ ) and  $s_i = 1$ . Once this is satisfied, all other  $IC$  constraints are satisfied automatically, by single crossing property of the utility function ( $\frac{\partial^2 u_i}{\partial x_i \partial \theta} > 0$ ).

An analogous logic ensues for any truth-telling player belonging to the set

---

<sup>12</sup>Given the nature of beta binomial distribution, signals are ex-ante correlated, and that given a signal, all possible contingencies of remaining  $(t-1)$  signals occur with a positive probability.

$1 - type$ . For a player  $i \in 1 - type$  to separate her messages in equilibrium, her equilibrium actions have to be  $< 1$  for every possible realization of the remaining  $(t - 1)$  signals. This further implies that the tightest  $IC$  in this case is one where the sufficient statistic of the rest of the truthful messages is the highest possible, ie,

$\sum_{j \in T \setminus \{i\}} s_j = (t - 1)$ , and  $s_i = 0$ . Again, as before, if this  $IC$  is satisfied, every other constraint would also be satisfied because of single crossing property.

Lemma 2 clearly shows the importance of alignment of interests for information transmission within an alliance. Despite varying degrees of interest about the ideal state of the world, it is possible for alliances to aggregate information as long as the dispersion in the biases is within a bound. To this effect, players in the group must be 'closely' aligned.  $A_i$  provides a measure of this alignment that ensures aggregation of information.

### 3.4 Conclusion

The full information aggregation result is useful for two reasons. Firstly, we introduce a novel methodology to obtain full information aggregation equilibrium with interdependent actions. The technique of using bounds on actions as  $IC$  constraints is, to the best of our knowledge, a first in the wider theoretical literature. Secondly, full aggregation result highlights the importance of alignment of interest for successful information sharing within members of an alliance. Specifically, we provide an intuitive characterization for cohesiveness of an alliance.

## Appendix A

# A Theory of Activism and Polarization

### A.1 Proofs - Benchmark model

All the proofs are carried out for candidate  $R$  and activist  $A_R$ , and are symmetric for candidate  $L$  and activist  $A_L$ .

#### A.1.1 Proof of Lemma 1

When platforms are not differentiated, irrespective of who wins the election, the ideological loss faced by the activist is constant and independent of  $c_R$ . As a result, there is no incentive to contribute since  $M'(0) > 0$ . Therefore, the only equilibrium of the activist contribution subgame is one where  $c_R = c_L = 0$ .

#### A.1.2 Proof of Lemma 2

Consider activist  $A_R$  and their contribution decision. Given candidate  $R$ 's win probability, the expected utility of activist  $A_R$  is,

$$EU_R^A(c_R; \beta) = K - \lambda(X_L - \beta)^2 - (1 - \lambda)(X_R - \beta)^2 - M(c_R)$$

Taking the first order condition, we get,

$$M'(c_R) - \frac{\eta}{4\sigma} P'(c_R)[2\beta - (X_L + X_R)] = 0$$

Rearranging, we get the necessary result.

$$\frac{M'(c_R)}{P'(c_R)} = \frac{\eta}{4\sigma} [2\beta - (X_L + X_R)] \quad (\text{A.1.1})$$

An analogous argument holds for the activist  $A_L$ .

$$\frac{M'(c_L)}{P'(c_L)} = \frac{\eta}{4\sigma} [2\beta + (X_L + X_R)] \quad (\text{A.1.2})$$

### A.1.3 Proof of Lemma 3

Under symmetric platforms,  $X_L + X_R = 0$  and from equation A.1.1, it must be that the optimal contribution by Activists  $A_R$  and  $A_L$  are equal. That is,  $c_R = c_L = c^*$ . Further, this  $c^*$  solves,

$$M'(c) = \frac{\eta\beta}{2\sigma} P'(c) \quad (\text{A.1.3})$$

For the second part of the lemma, we apply implicit function theorem to equation A.1.3.

$$\text{Let } \Psi = \frac{\eta\beta}{2\sigma} P'(c) - M'(c)$$

$$\frac{dc^*}{d\beta} = -\frac{\frac{\partial \Psi}{\partial \beta}}{\frac{\partial \Psi}{\partial c}} = -\frac{\eta}{2\sigma} \frac{P'(c)}{\frac{\eta\beta}{2\sigma} P''(c) - M''(c)}$$

Since  $P''(\cdot) \leq 0$  and  $M''(c) > 0$ , it follows that  $\frac{dc^*}{d\beta} > 0$ .

Similarly,  $\frac{dc^*}{d\eta} = -\frac{\beta}{2\sigma} \frac{P'(c)}{\frac{\eta\beta}{\sigma} P''(c) - M''(c)} > 0$  and  $\frac{dc^*}{d\sigma} = \frac{\eta\beta}{2\sigma^2} \frac{P'(c)}{\frac{\eta\beta}{\sigma} P''(c) - M''(c)} < 0$  for the same arguments.

### A.1.4 Proof of Proposition 1

When  $\eta = 0$ , the win probability of candidate  $R$  is just  $1 - \lambda = \frac{1}{2} - \frac{(X_R + X_L)}{4\sigma}$ . Each candidate chooses a platform to maximize their payoffs, taking as given the platform chosen by the other candidate.



The SPNE is such a pair of platform choices that maximizes the expected utility of both the candidates in the first stage. Let us consider candidate R.

$$EU_R^C(X_L, X_R) = -\lambda(X_L - \alpha)^2 - (1 - \lambda)(X_R - \alpha)^2 + (1 - \lambda)b$$

Supposing that candidate L chooses  $X_L = -x$ . Taking the FOC and evaluating the expression at  $(-x, x)$ ,

$$\frac{dEU_R^C(-x, X_R)}{dX_R} \big|_{(-x, x)} = \lambda'_{X_R} [4\alpha x + b] - (\alpha - x) = 0 \text{ where } \lambda'_{X_R} = \frac{1}{4\sigma}$$

Solving this gives us  $\bar{x} = \frac{4\sigma\alpha - b}{4(\alpha + \sigma)}$ . If  $\alpha > \frac{b}{4\sigma}$ , then  $\bar{x} > 0$  follows from the expression.

### A.1.5 Proof of Proposition 2

In the case of  $\eta > 0$ , the win-probability is affected by activism.

$$\text{That is, } \lambda'_{X_R} = \frac{1}{4\sigma} + \frac{\eta}{4\sigma} \left[ \frac{(X_R - X_L)[P'(c_L) \cdot \frac{dc_L}{dX_R} - P'(c_R) \frac{dc_R}{dX_R}] - [P(c_L) - P(c_R)]}{(X_R - X_L)^2} \right].$$

Applying implicit function theorem to equations A.1.1 and A.1.2, we can compute

$\frac{dc_R}{dX_R}$  and  $\frac{dc_L}{dX_R}$  respectively.

$$\begin{aligned} \frac{dc_R}{dX_R} &= \frac{\eta}{4\sigma} \frac{P'(c_R)}{\frac{\eta}{4\sigma}(2\beta - (X_L + X_R))P''(c_R) - M''(c_R)} \\ \frac{dc_L}{dX_R} &= -\frac{\eta}{4\sigma} \frac{P'(c_L)}{\frac{\eta}{4\sigma}(2\beta + (X_L + X_R))P''(c_L) - M''(c_L)} \end{aligned}$$

At  $(-x, x)$ , it is easy to check that  $\frac{dc_R}{dX_R} \big|_{(-x, x)} = -\frac{dc_L}{dX_R} \big|_{(-x, x)}$  and,

$$\frac{dc_R}{dX_R} \big|_{(-x, x)} = \frac{\eta}{4\sigma} \frac{P'(c)}{\frac{\eta\beta}{2\sigma}P''(c) - M''(c)}$$

Therefore, the FOC evaluated at  $(-x, x)$  gives us,

$$\begin{aligned} \lambda'_{X_R} &= \frac{1}{4\sigma} - \frac{\eta^2}{16\sigma^2 x} \frac{(P'(c))^2}{\frac{\eta\beta}{2\sigma}P''(c) - M''(c)} \\ \lambda'_{X_R} &= \frac{1}{4\sigma} \left[ 1 - \frac{\eta^2}{4\sigma x} \frac{(P'(c))^2}{\frac{\eta\beta}{2\sigma}P''(c) - M''(c)} \right] \end{aligned}$$

But we know that  $c$  solves  $M'(c) = \frac{\eta\beta}{2\sigma}P'(c)$ . Simplifying the above expression and using the fact that  $\gamma_m(c) = c \frac{M''}{M'}$  and  $\gamma_p(c) = -c \frac{P''}{P'}$ ,

$$\Rightarrow \lambda'_{X_R} = \frac{1}{4\sigma} \left[ 1 + \frac{\eta}{2\beta x} \frac{cP'(c)}{\gamma_m(c) + \gamma_p(c)} \right] > 0$$

Rewriting the above as  $\lambda'_{X_R} = \frac{1}{4\sigma} [1 + \frac{1}{2x} D(c, \eta, \beta)]$  and using this in the FOC evaluated at  $(-x, x)$ ,

$$\lambda'_{X_R}[4\alpha x + b] - (\alpha - x) = 0$$

This yields us the required condition,

$$4(\alpha + \sigma)x^2 - [4\alpha(\sigma - \frac{1}{2}D(c, \eta, \beta)) - b]x + \frac{b}{2}.D(c, \eta, \beta) = 0 \quad (\text{A.1.4})$$

where  $D(c, \eta, \beta) = \frac{\eta}{\beta} \cdot \frac{c.P'(c)}{\gamma_m(c) + \gamma_p(c)}$

$x = 0$  if  $4\alpha(\sigma - \frac{1}{2}D(c, \eta, \beta)) - b < 0$

This implies that  $\alpha \leq \frac{b}{4(\sigma - \frac{1}{2}D(.))}$  for  $x^* = 0$ .

Further, notice that  $x$  is decreasing in  $D(.)$ . As a result, when  $\eta = 0$  and  $D(.) = 0$  the resulting equilibrium  $\bar{x} > x_{D(.)>0}$ . This proves the proposition.

### A.1.6 Proof of Proposition 3

From the equilibrium equation A.1.4, let  $\phi = 4(\alpha + \sigma)x^2 - [4\alpha(\sigma - \frac{1}{2}D(c, \eta, \beta)) - b]x + \frac{b}{2}.D(c, \eta, \beta)$ . Then, the following holds:  $\frac{\partial \phi}{\partial x} > 0$  at the equilibrium  $(-x, x)$ . To see this,

$$\frac{d\phi}{dx} = 8(\alpha + \sigma)x - [4\alpha(\sigma - \frac{1}{2}D(c, \eta, \beta)) - b]$$

That is,  $\frac{d\phi}{dx} > 0$  iff  $x > \frac{[4\alpha(\sigma - \frac{1}{2}D(c, \eta, \beta)) - b]}{8(\alpha + \sigma)}$ . But,

$$x = \frac{1}{8(\alpha + \sigma)} [[4\alpha(\sigma - \frac{1}{2}D(c, \eta, \beta)) - b] + \sqrt{[4\alpha(\sigma - \frac{1}{2}D(c, \eta, \beta)) - b]^2 - 8b(\alpha + \sigma)D(c, \eta, \beta)}]$$

$$\Rightarrow x = \frac{[4\alpha(\sigma - \frac{1}{2}D(c, \eta, \beta)) - b]}{8(\alpha + \sigma)} + \frac{\sqrt{[4\alpha(\sigma - \frac{1}{2}D(c, \eta, \beta)) - b]^2 - 8b(\alpha + \sigma)D(c, \eta, \beta)}}{8(\alpha + \sigma)}$$

Therefore it holds that  $\frac{d\phi}{dx} > 0$ . Given this,

$$\frac{dx}{d\beta} = -\frac{\frac{d\phi}{d\beta}}{\frac{d\phi}{dx}}$$

This implies that  $\frac{dx}{d\beta} > 0$  if  $\frac{d\phi}{d\beta} < 0$ .

$$\frac{d\phi}{d\beta} = \frac{d\phi}{dD} \frac{dD(\cdot)}{d\beta} = (4\alpha x + b) \frac{dD(\cdot)}{d\beta}$$

$$\frac{dD(\cdot)}{d\beta} = \frac{\partial D(\cdot)}{\partial c} \frac{\partial c}{\partial \beta} + \frac{\partial D(\cdot)}{\partial \beta}$$

where,

$$\frac{\partial c}{\partial \beta} = -\frac{\eta}{\sigma} \frac{P'(c)}{\frac{\eta\beta}{\sigma} P''(c) - M''(c)} = \frac{1}{\beta} \frac{c}{(\gamma_m(c) + \gamma_p(c))}$$

$$\Rightarrow \frac{\partial c}{\partial \beta} = \frac{D(\cdot)}{\eta P'(c)}$$

$$\frac{\partial D(\cdot)}{\partial \beta} = -\frac{\eta}{\beta^2} \cdot \frac{c \cdot P'(c)}{\gamma_m(c) + \gamma_p(c)} = -\frac{1}{\beta} D(\cdot)$$

$$\frac{\partial D(\cdot)}{\partial c} = \frac{\eta}{\beta} \left[ \frac{(\gamma_m(c) + \gamma_p(c))(cP'' + P') - cP'(\gamma'_m(c) + \gamma'_p(c))}{(\gamma_m(c) + \gamma_p(c))^2} \right]$$

$$\frac{dD(\cdot)}{d\beta} = D(\cdot) \left[ \frac{1}{\eta P'(c)} \frac{\partial D(\cdot)}{\partial c} - \frac{1}{\beta} \right]$$

$$\text{Therefore, } \frac{dD(\cdot)}{d\beta} > 0 \text{ iff } \frac{\partial D(\cdot)}{\partial c} > \frac{\eta P'(c)}{\beta}.$$

$$\Rightarrow \frac{(\gamma_m(c) + \gamma_p(c))(cP'' + P') - cP'(\gamma'_m(c) + \gamma'_p(c))}{(\gamma_m(c) + \gamma_p(c))^2} > P'$$

Simplifying the above equation implies that  $\frac{dD(\cdot)}{d\beta} > 0$  when

$$\frac{1 - \gamma_p(c)}{\gamma_m(c) + \gamma_p(c)} - c \frac{\gamma'_m(c) + \gamma'_p(c)}{(\gamma_m(c) + \gamma_p(c))^2} > 1.$$

However,  $\frac{dD(\cdot)}{d\beta} > 0 \Rightarrow \frac{d\phi}{d\beta} > 0 \Rightarrow \frac{dx}{d\beta} < 0$ . Moreover, when  $\gamma'_m(c) = \gamma'_p(c) = 0$ , the

above condition simplifies to,

$$\frac{1 - \gamma_p(c)}{\gamma_m(c) + \gamma_p(c)} > 1 \Rightarrow \gamma_p(c) < \frac{1 - \gamma_m(c)}{2}$$

A similar argument follows for the other two cases. This completes the proof.

### A.1.7 Proof of Proposition 4

The comparative statics result with respect to  $\alpha$  and  $b$  follows from Bernhardt, Duggan and Squintani [19]. I will therefore concentrate on the parameters  $\eta$  and  $\sigma$ .

As before, the sign of  $\frac{dD(\cdot)}{d\eta}$  determines the sign of  $\frac{dx}{d\eta}$ .

$$\frac{dD(\cdot)}{d\eta} = \frac{\partial D(\cdot)}{\partial c} \frac{\partial c}{\partial \eta} + \frac{\partial D(\cdot)}{\partial \eta}$$

Since  $\frac{\partial c}{\partial \eta}, \frac{\partial D(\cdot)}{\partial \eta} > 0$ , and  $\frac{\partial D(\cdot)}{\partial c} > 0$ , it must be that  $\frac{dD(\cdot)}{d\eta} > 0$  which further implies

that  $\frac{dx}{d\eta} < 0$ .

$$\frac{dD(\cdot)}{d\sigma} = \frac{\partial D(\cdot)}{\partial c} \frac{\partial c}{\partial \sigma} + \frac{\partial D(\cdot)}{\partial \sigma} = \frac{\partial D(\cdot)}{\partial c} \frac{\partial c}{\partial \sigma}$$

Since  $\frac{\partial c}{\partial \sigma} < 0$ , this implies that  $\frac{dD(\cdot)}{d\sigma} < 0$  and  $\frac{dx}{d\sigma} > 0$ . This completes the proof.

### A.1.8 Proof of Lemma 4

$$W_{mv}(-x, x) = - \int_{-\sigma}^0 (x + \mu)^2 d\mu - \int_0^{\sigma} (x - \mu)^2 d\mu + \eta P(c^*)$$

$$W_{mv}(-x, x) = \eta P(c^*) - [2\sigma x^2 - 2\sigma^2 x + \frac{2}{3}\sigma^3]$$

$$\text{i) } \frac{dW_{mv}(-x, x)}{dx} = 2\sigma^2 - 4\sigma x > 0 \text{ at } (0, 0)$$

$$\text{ii) } \frac{dW_{mv}(-x, x)}{dx} = 0 \text{ at } (-\frac{\sigma}{2}, \frac{\sigma}{2}).$$

Substituting this into the above welfare equation gives

$$W_{mv}(-\frac{\sigma}{2}, \frac{\sigma}{2}) = \eta P(c^*) - \frac{1}{6}\sigma^3$$

### A.1.9 Proof of Lemma 5

Total welfare is given by:

$$W_{tot}(-x, x) = \eta P(c^*) - [2\sigma x^2 - 2\sigma^2 x + \frac{2}{3}\sigma^3] + 2 [K - x^2 - \beta^2 - M(c^*)]$$

$$\frac{dW_{tot}(-x, x)}{dx} = 0 \text{ if } 2\sigma^2 - 4\sigma x - 4x = 0$$

This gives us  $x^{so} = \frac{\sigma}{\sigma+1} \frac{\sigma}{2} < x^{vo}$ . Any equilibrium  $x \in (0, x^{vo})$  is pareto optimal since the activist and the voter have diverging preferences, meaning that a voter always prefers a higher  $x$  in this interval whereas the activist prefers the convergence to  $(0, 0)$ . This implies that given an equilibrium in this range, any deviation is going to hurt either the voter or the activist, and therefore, is pareto optimal. Finally, the last part of the proof follows from Proposition 6 (pg. 577) in Bernhardt, Duggan and Squintani [19]. A similar argument ensues for  $\eta$ .

### A.1.10 Proof of Proposition 5

When  $x < x^{so}$ , the overall welfare is increasing in  $x$  from the concavity of the welfare function. As a result, when the  $WTE$  is high then  $\frac{dx}{d\beta} < 0$ . This means that  $\frac{dW_{tot}(-x, x)}{d\beta} = \frac{dW_{tot}(-x, x)}{dx} \cdot \frac{dx}{d\beta} < 0$ . The opposite holds when  $WTE$  is low. Above  $x^{so}$ , the overall welfare is decreasing in  $x$  ( $\frac{dW_{tot}(-x, x)}{dx} < 0$ ). Hence  $\frac{dW_{tot}(-x, x)}{d\beta} = \frac{dW_{tot}(-x, x)}{dx} \cdot \frac{dx}{d\beta} > 0$  when the  $WTE$  parameter is high. This completes the proof.

### A.1.11 Proof of Proposition 6

Under Assumption 1,  $W_{act}(0, 0) > W_{mv}(0, 0)$  and  $W_{act}(-\sigma, \sigma) < W_{mv}(-\sigma, \sigma)$ . Further, when Assumption 2 is satisfied, it implies that  $W_{mv}(-\frac{\sigma}{2}, \frac{\sigma}{2}) \geq W_{act}(-\frac{\sigma}{2}, \frac{\sigma}{2})$ . This, combined with the fact that  $\frac{dW_{act}(-x, x)}{dx} < 0$  and  $\frac{dW_{mv}(-x, x)}{dx} > 0$  in the interval  $(0, \frac{\sigma}{2})$  implies that there must exist a unique  $x^{na}$  such that  $W_{act}(x^{na}, x^{na}) = W_{mv}(x^{na}, x^{na})$ .

To show the next part, observe that fixing any  $(-x, x)$ ,  $\frac{dW_{mv}(-x, x)}{d\beta} > 0$  and  $\frac{dW_{act}(-x, x)}{d\beta} < 0$ . That is the welfare function of voters shifts up while that of activists shifts down as the partisanship increases. This implies that, by single crossing property of the welfare functions, the point where the welfare is equal must be lower than the previous equilibrium. To see this, take  $(\beta_1, x_1^{na})$  and  $(\beta_2, x_2^{na})$  be two pairs. Suppose that  $\beta_2 > \beta_1$  and also that  $x_2^{na} > x_1^{na}$ . We know that  $W_{mv}^{\beta_1}(-x_1^{na}, x_1^{na}) = W_{act}^{\beta_1}(-x_1^{na}, x_1^{na})$  and  $W_{mv}^{\beta_2}(-x_2^{na}, x_2^{na}) = W_{act}^{\beta_2}(-x_2^{na}, x_2^{na})$ .

But, since  $\frac{dW_{mv}(-x, x)}{d\beta} > 0$  and  $\frac{dW_{act}(-x, x)}{d\beta} < 0$ , the following ordering must be satisfied:

$$W_{mv}^{\beta_2}(-x_1^{na}, x_1^{na}) > W_{mv}^{\beta_1}(-x_1^{na}, x_1^{na}) = W_{act}^{\beta_1}(-x_1^{na}, x_1^{na}) > W_{act}^{\beta_2}(-x_1^{na}, x_1^{na})$$

This violates the definition of the equilibrium  $x^{na}$ . Therefore  $x_1^{na} > x_2^{na}$  whenever  $\beta_2 > \beta_1$ .

The last part of the proof follows from the earlier observation that

$\frac{dW_{act}(-x, x)}{dx} < 0$  and  $\frac{dW_{mv}(-x, x)}{dx} > 0$  in the interval  $(0, \frac{\sigma}{2})$ . This implies that when  $x < x^{na}$ ,

$$W_{mv}(-x^{na}, x^{na}) > W_{mv}(-x, x) \text{ and } W_{act}(-x^{na}, x^{na}) < W_{act}(-x, x)$$

Hence,  $W_{act}(-x, x) - W_{mv}(-x, x) > 0$  when  $x < x^{na}$ . This completes the proof.

### A.1.12 Proof of Proposition 7

The optimal  $K^*$  can be solved for directly. By its definition,  $K^*$  solves the following:

$$W_{act}(-x^{so}, x^{so}) = W_{mv}(-x^{so}, x^{so}). \text{ This yields us}$$

$$K^* = \frac{1}{2}[\eta P(c^*) + 2M(c^*)] + [\beta^2 - \frac{(2\sigma^2 + \sigma + 8)}{24(\sigma + 1)^2}] \quad (\text{A.1.5})$$

Further, since  $c^*$  is increasing in both  $\beta$  and  $\eta$ ,  $\frac{dK^*}{d\beta} > 0$  and  $\frac{dK^*}{d\eta} > 0$ .

Also, observe that increasing  $K$  above this threshold implies that  $x^{na} > x^{so}$  since  $\frac{dW_{act}(-x, x)}{dK} > 0$ . As a consequence, implementing the social optimal provides the activist with a welfare surplus (Proposition 6). This completes the proof.

## A.2 Proofs - Extensions

### A.2.1 Activism and Noisy Campaigns

Suppose, for sake of exposition,  $\sigma = 1$ . The solution to the electoral model is solved backwards, as previously. As before, we present the results for candidate (and activist)  $R$ .

The median voter observes the platforms with a noise:  $\tilde{X}_L = X_L + \eta_L$ ,  $\tilde{X}_R = X_R + \eta_R$ .

Voter prefers candidate  $L$  if,  $E[-(\tilde{X}_L - \mu)^2] \geq E[-(\tilde{X}_R - \mu)^2]$

$$\Leftrightarrow E[(X_L - \mu)^2 + \eta_L^2 + 2\eta_L(X_L - \mu)] \leq E[(X_R - \mu)^2 + \eta_R^2 + 2\eta_R(X_R - \mu)]$$

$$\Leftrightarrow E(\eta_L^2) - E(\eta_R^2) \leq (X_R - \mu)^2 - (X_L - \mu)^2$$

$$\Leftrightarrow a(c_L) - a(c_R) \leq (X_R - \mu)^2 - (X_L - \mu)^2$$

$$\mu \leq \frac{(X_R + X_L)}{2} + \frac{a(c_R) - a(c_L)}{2(X_R - X_L)}$$

The win-probability for Candidate  $L$  is  $\lambda = \frac{1}{2} + \frac{(X_R + X_L)}{4} + \frac{a(c_R) - a(c_L)}{4(X_R - X_L)}$ . As in the

original model, the the equilibrium condition for participation is

$$\frac{M'(c_R)}{a'(c_R)} = \frac{[X_R + X_L - 2\beta]}{4} \quad (\text{A.2.1})$$

By symmetry, the equivalent condition for activist  $L$  is,

$$\frac{M'(c_L)}{a'(c_L)} = -\frac{[X_R + X_L + 2\beta]}{4} \quad (\text{A.2.2})$$

At the symmetric equilibrium  $(-x, x)$ , equations A.2.1 and A.2.2 reduce to  $\frac{M'(c)}{a'(c)} = -\frac{\beta}{2}$ . Let  $D(c(\beta), \beta) = -\frac{1}{\beta} \cdot \frac{ca'(c)}{\gamma_m + \gamma_n} \approx D$ . Then, the equation that solves for the symmetric equilibrium is given by the following:

$$4(\alpha + 1)x^2 - (4\alpha - b - 2\alpha D)x + \frac{b}{2}D = 0 \quad (\text{A.2.3})$$

$$\frac{M'(c)}{a'(c)} = -\frac{\beta}{2} \quad (\text{A.2.4})$$

Together, the above two equations determine the symmetric equilibrium platform of candidates, and mobilization by activists. The dependence of equilibrium platform  $x$  on  $\alpha$  and  $b$  are along the lines of the main model. To derive comparative statics with respect to  $\beta$ , let  $\Phi = 4(\alpha + 1)x^2 - (4\alpha - b - 2\alpha D)x + \frac{b}{2}D$ .

$$\begin{aligned} \frac{dx}{d\beta} &= -\frac{\frac{d\Phi}{d\beta}}{\frac{d\Phi}{dx}} = -\frac{\frac{d\Phi}{d\beta}}{+ve} \\ \frac{d\Phi}{d\beta} &= \frac{d\Phi}{dD} \left[ \frac{\partial D}{\partial \beta} + \frac{\partial D}{\partial c} \frac{\partial c}{\partial \beta} \right] \end{aligned}$$

The rest of the proof follows from arguments similar to one in Proposition 3. The sign of  $\frac{\partial \Phi}{\partial \beta}$  depends on the expression  $(-2 + \frac{1+\gamma_m}{\gamma_m + \gamma_n})$ , which is greater than zero when  $\gamma_n < \frac{1-\gamma_m}{2}$ . When this is satisfied,  $\frac{\partial \Phi}{\partial \beta} > 0$  and further,  $\frac{\partial x}{\partial \beta} < 0$ . When the sign of this inequality is reversed, that is  $\gamma_n > \frac{1-\gamma_m}{2}$ , then  $\frac{\partial x}{\partial \beta} > 0$ . This concludes the analysis.

### A.2.2 The role of big money in campaigns

The participation in equilibrium is modified to

$$M'(c) = \frac{\eta\beta}{2\sigma} \cdot P'_2(S, c)$$

Applying implicit function theorem yields,  $\frac{dc}{dS} = -\frac{\frac{\eta\beta}{2\sigma} P''_{12}(S, c)}{\frac{\eta\beta}{2\sigma} \cdot P''_{22}(S, c) - M''(c)} < 0$ .

Similarly, the equilibrium is just the modified equation where,

$$D(c, \eta, \beta, S) = \frac{\eta}{\beta} \cdot \frac{c \cdot P'_2(S, c)}{\gamma_m(c) + \gamma_p(c)}$$

The sign of  $\frac{dD}{dS}$  determines the sign of  $\frac{dx}{dS}$ , as before.

$$\frac{dD}{dS} = \frac{\partial D(\cdot)}{\partial c} \frac{\partial c}{\partial S} + \frac{\partial D(\cdot)}{\partial S}$$

Since  $\frac{\partial D(\cdot)}{\partial S} < 0$  and  $\frac{\partial c}{\partial S} < 0$ , and from our earlier assumption on  $WTE$ , it is true that  $\frac{dD}{dS} < 0$ . This further implies that  $\frac{dx}{dS} > 0$ .



## Appendix B

# Cheap Talk and Strategic Substitutability

### B.1 Formal definitions of PBE and PRE

#### B.1.1 PBE:

A communication equilibrium of the game is given by:

1. given the message  $m$  and posterior beliefs  $p(\theta \mid m)$ ,  $R$  and  $S$  simultaneously choose actions  $(x_R^*(m), x_S^*(\theta, m))$  that maximizes their expected utility according to the following dual (constrained) optimization problem:

$$\begin{aligned} \max_{x_R \in V} E_\theta[U(\phi^R(x_R, x_S^*(\theta, m)), \theta)] \text{ subject to } x_S^*(\theta, m) \equiv \\ \max_{x_S \in V} U(\phi^S(x_S, x_R^*(m)), \theta, b), x_S \in V \end{aligned} \quad (\text{B.1.1})$$

$$\begin{aligned} \max_{x_S \in V} E_\theta[U(\phi^S(x_S, x_R^*(m)), \theta, b)] \text{ subject to } x_R^*(m) \equiv \\ \arg \max_{x_R \in V} U(\phi^R(x_R, x_S^*(\theta, m)), \theta), x_R \in V \end{aligned} \quad (\text{B.1.2})$$

2. the optimal actions by the two players in equilibrium,  $x_R^*(m)$  and  $x_S^*(\theta, m)$ ,

together, must ensure that the contribution function maximizes each players' expected utility conditional on their information, ie,  $\phi^{R*}(x_R^*(m), x_S^*(\theta, m)) \equiv \arg \max_{\phi^R} U(\phi^R(x_R, x_S), \theta)$  and  $\phi^{S*}(x_S^*(\theta, m), x_R^*(m)) \equiv \arg \max_{\phi^S} U(\phi^S(x_S, x_R), \theta, b)$

3. the posterior beliefs,  $p(\cdot | m)$ , are updated using Bayes' rule whenever possible, given the messaging rule  $\mu(\cdot)$
4. given the beliefs and second stage contributions  $x_R(m)$  and  $x_S(\theta, m)$ ,  $S$  chooses a reporting strategy that maximises expected payoff in the first stage,

$$\mu(\theta) \in \arg \max_{m \in M} U(\phi^S(x_S(\theta, m), x_R(m)), \theta, b)$$

### B.1.2 PRE:

The PRE of the game consists of a  $\theta^*$  such that,

- for all  $\theta \leq \theta^*$ ,  $S$  sends a separating (truthful) message  $m = \theta$ ; for every  $\theta > \theta^*$ ,  $S$  sends a pooling message  $m = 1$
- When  $m \leq \theta^*$ , the posterior beliefs are  $p(\theta | m = \theta) = 1$ ; when  $m = 1$ ,  $p(\theta | m) = f(\theta | \theta > \theta^*)$
- Upon receiving message  $m \leq \theta^*$ , players' optimal actions are  $x_S^*(m) = \tilde{x}_S(m)$  and  $x_R^*(m) = \tilde{x}_R(m)$
- Upon receiving message  $m = 1$ , players' optimal actions are,
$$\tilde{x}_S(\theta, (\theta^*1]) \equiv \arg \max_{x_S \in \mathbb{R}} \int_{\theta^*}^1 U(\phi^S(x_S, x_R^*((\theta^*, 1])), \theta, b) f(\theta | \theta > \theta^*) d\theta$$
and
$$x_R^*((\theta^*, 1]) \equiv \arg \max_{x_R \in V} \int_{\theta^*}^1 U(\phi^R(x_R, \tilde{x}_S(\theta, (\theta^*1])), \theta) f(\theta | \theta > \theta^*) d\theta$$
  - If  $\tilde{x}_S(\theta, (\theta^*1]) \leq c$ , then  $x_S^*(\theta, (\theta^*1]) = \tilde{x}_S(\theta, (\theta^*1])$  and  $x_R^*((\theta^*, 1])$  is as above

$$\begin{aligned}
& - \text{ If } \tilde{x}_S(\theta, (\theta^*1]) > c, \text{ then } x_S^*(\theta, (\theta^*1]) = c \text{ and } x_R^*((\theta^*, 1]) \equiv \\
& \arg \max_{\theta^*} \int_{\theta^*}^1 U(\phi^R(x_R, c), \theta) f(\theta \mid \theta > \theta^*) d\theta
\end{aligned}$$

The first condition says that for all states in  $[0, \theta^*]$ ,  $S$  communicates truthfully, and for any state above, pools by sending an exaggerated message  $m = 1$ . The second condition describes the formation of posterior beliefs. For any message on  $[0, \theta^*]$ ,  $R$  believes it to be truthful and for messages  $m = 1$ , the posterior is just the conditional prior on the state space.

The third and fourth statements indicate the equilibrium actions conditional on the message and the posterior beliefs of  $R$ . The important departure to note is when the sender's unconstrained action goes above the bound. Specifically,  $R$  best responds by taking into account the possibility that  $S$ 's action is constrained by the upper bound -  $\hat{x}_S(\theta, m) > c$  implies  $x_S^*(\theta, m) = c$ - and adjusts her actions accordingly. This revised best response is indicated by the last sub-condition.

## B.2 Proofs

### B.2.1 Proof of Proposition 9

*Communication Breakdown:*

When LTIS is violated,  $\tilde{x}_S(0) > c$ , meaning that the unconstrained actions does not coincide with the constrained optimization action  $x_S^*(0) = c$ . Moreover, since  $U_{12} > 0$ , it must hold for every  $\theta \in [0, 1]$  that  $\tilde{x}_S(\theta) > c$ . But if this is so, truth-telling can never be optimal since the sender can always inflate her message and ensure  $R$  contributes more. To see this point, think of a generic  $\theta'$ . If  $S$  reports  $\theta'$ , the optimal actions are  $x_S^*(\theta') = c$  and  $x_R^*(\theta')$  solves  $\max_{x_R \in V} U(\phi^R(x_R, c), \theta')$ . However, this  $x_R^*(\theta')$  is inefficient since  $\phi^{R^*}(x_R^*(\theta'), c) < \phi^{S^*}(c, x_R^*(\theta')) < \tilde{\phi}^S(\tilde{x}_S(\theta'), \tilde{x}_R(\theta'))$ . The term  $\tilde{\phi}^S(\tilde{x}_S(\theta'), \tilde{x}_R(\theta'))$  is the optimal contribution function that maximizes  $S$ 's payoff. But, because LTIS is violated, the contribution under truth-telling is  $\phi^{S^*}(c, x_R^*(\theta'))$  which is clearly sub-optimal in the sense that  $U_1(\phi^{S^*}(c, x_R^*(\theta')), \theta', b) >$

0. This condition corresponds with the '*positive spillover effect at the bound*'. Finally, note that  $\phi_2^{S*}(c, x_R^*(\theta')) \geq 0$  by assumption 2. This implies that if  $S$  instead sends a higher message  $\theta'' > \theta'$ ,  $R$  increases her optimal action thereby improving  $S$ 's payoff. Of course, anticipating this,  $R$  must never believe any message  $m$  as being truthful. The same argument can be applied for any  $\theta \in [0, 1)$ . Hence, this leads to a communication breakdown with all types pooling on the message  $m = 1$ .

*Full revelation result:*

When HTIS condition is satisfied, it implies that for every  $\theta \in [0, 1)$ ,  $\tilde{x}_S(\theta) < c$  by single crossing property of the utility function ( $U_{12} > 0$ ). But if this is the case, when the sender sends a truthful message  $m = \theta$ , the optimal action under both constrained optimization and unconstrained optimization coincide for the sender. This means that for every  $\theta \in [0, 1]$ ,  $x_S^*(\theta) = \tilde{x}_S(\theta)$ . This ensures there is no inefficiency in terms of contributions. Hence, there always exists a full revelation equilibrium in which the sender has an incentive to reveal her information truthfully.

### B.2.2 Proof of Proposition 10

For  $\theta^* = \bar{\theta}$  to be supported as an equilibrium, I will construct the following off-equilibrium path beliefs: for any  $m \in (\bar{\theta}, 1)$ ,  $R$  assigns the belief  $\theta = \bar{\theta}$ , that is the deviation comes from the lowest possible type in the set of possible off-equilibrium path messages. Then, for an equilibrium with cutoff  $\bar{\theta}$  to exist, there should be no profitable deviations for any of the types of players. To check this, consider the types of in  $(0, \bar{\theta}]$  and  $(\bar{\theta}, 1]$ . For any  $\theta \in (0, \theta^*]$ ,  $S$  does not have an incentive to deviate from truth telling since the optimal action under unconstrained optimization coincides with the constrained optimization problem, implying that  $\tilde{x}_S(\theta) = x_S^*(\theta) \leq c$ .

For  $\theta \in (\bar{\theta}, 1]$ , however,  $\tilde{x}_S(\theta, (\bar{\theta}, 1]) \leq c$  or  $\tilde{x}_S(\theta, (\bar{\theta}, 1]) > c$ . Since  $\tilde{x}_S(\bar{\theta}, (\bar{\theta}, 1]) < c$  and  $\tilde{x}_S(1, (\bar{\theta}, 1]) > c$ , from single crossing, continuity of  $U(\cdot)$  and  $\phi^S(\cdot)$  and assumption 5, there must exist a  $\theta' \in (\bar{\theta}, 1)$  such that  $\tilde{x}_S(\theta', (\bar{\theta}, 1]) = c$ . Then, for every  $\theta$  in  $(\bar{\theta}, \theta']$ , it must hold true that  $\tilde{x}_S(\theta, (\bar{\theta}, 1]) \leq c$ . Therefore, sending the

message  $m = 1$  maximizes payoff for the same reasons put forth above. For types in  $(\theta', 1]$ , the constrained optimization solution suggests that  $S$ 's action hits the bound, meaning  $x_S^*(\theta, (\bar{\theta}, 1]) = c$ . But, the payoff to  $S$  from sending  $m = 1$  is still higher than sending any other off-equilibrium path message. To see this, notice that  $x_R^*((\bar{\theta}, 1]) > x_R^*(\bar{\theta})$  and  $\phi_2^{S*}(c, x_R^*(.)) > 0$  at the bound. Therefore,  $U(\phi_2^{S*}(c, x_R^*((\bar{\theta}, 1])), \theta, b) \geq U(\phi_2^{S*}(c, x_R^*(\bar{\theta})), \theta, b)$  for all  $\theta \in (\theta', 1]$  such that  $x_S^*(\theta, (\bar{\theta}, 1]) = c$ . This concludes the proof.

### B.2.3 Proof of Proposition 11

I will start with defining the off-equilibrium path messages that would be sufficient to support a PRE. Specifically, for any  $m \in (\theta^*, 1)$ ,  $R$  assigns the belief  $\theta = \theta^*$ , that is the deviation comes from the lowest possible type in set of possible off-equilibrium path messages. Then, for an equilibrium with  $\theta^*$  to exist, there should be no profitable deviations for any of the types of players. To check this, consider the types of in  $(0, \theta^*]$  and  $(\theta^*, 1]$ , in that order. For any  $\theta \in (0, \theta^*]$ ,  $S$  does not have an incentive to deviate from truth telling since the optimal action upon truthful messaging is strictly within the bound,  $\tilde{x}_S(\theta) < c$ . This implies that the unconstrained solution also coincides with the constrained optimization problem,  $\tilde{x}_S(\theta) = x_S^*(\theta)$ .

For  $\theta \in (\theta^*, 1]$ ,  $\tilde{x}_S(\theta, (\theta^*, 1]) \leq c$  or  $\tilde{x}_S(\theta, (\bar{\theta}, 1]) > c$ . Since  $\tilde{x}_S(\theta^*, (\theta^*, 1]) < c$  and  $\tilde{x}_S(1, (\theta^*, 1]) > c$ , as before, from single crossing, continuity of  $U(.)$  and  $\phi^S(.)$  and assumption 5, there must exist a  $\theta' \in (\theta^*, 1)$  such that  $\tilde{x}_S(\theta', (\theta^*, 1]) = c$ . Then, for every  $\theta$  in  $(\theta^*, \theta']$ , it must hold true that  $\tilde{x}_S(\theta, (\theta^*, 1]) \leq c$ . For the types  $\theta \in (\theta', 1]$ , the constrained optimization solution suggests that  $S$ 's action hits the bound, meaning  $x_S^*(\theta, (\theta^*, 1]) = c$ . But, the payoff to  $S$  from sending  $m = 1$  is still higher than sending any other off-equilibrium path message. To see this, notice that  $x_R^*((\bar{\theta}, 1]) > x_R^*(\bar{\theta})$  and  $\phi_2^{S*}(c, x_R^*(.)) > 0$  at the bound. Therefore,  $U(\phi_2^{S*}(c, x_R^*((\bar{\theta}, 1])), \theta, b) \geq U(\phi_2^{S*}(c, x_R^*(\bar{\theta})), \theta, b)$  for all  $\theta \in (\theta', 1]$  such that  $x_S^*(\theta, (\bar{\theta}, 1]) = c$ . This completes the proof.

## B.2.4 Proof of Proposition 12

**Receiver's ex-ante utility:**

$$V_R(\theta^*) = \int_0^{\theta^*} U(\phi^{R^*}(x_R^*(t), x_S^*(t)), t) f(t) dt + \int_{\theta^*}^1 U(\phi^{R^*}(x_R^*((\theta^*, 1]), x_S^*(t, (\theta^*, 1])), t) f(t) dt$$

Taking the derivative of receiver's welfare with respect to  $\theta^*$ ,

$$\frac{dV_R(\theta^*)}{d\theta^*} = U(\phi^{R^*}(x_R^*(\theta^*), x_S^*(\theta^*)), \theta^*) f(\theta^*) - U(\phi^{R^*}(x_R^*((\theta^*, 1]), x_S^*(\theta^*, (\theta^*, 1])), \theta^*) f(\theta^*) > 0$$

for any  $\theta^* \leq \bar{\theta}$

**Sender's ex-ante utility:**

Take any two feasible cutoffs of a PRE, say  $\theta'$  and  $\theta''$ , such that  $\theta' < \theta'' \leq \bar{\theta}$ . I will try to establish that sender is better off with the more informative equilibrium  $\theta''$ . First, observe that by assumptions of continuity, for any cutoff equilibrium  $\theta^*$  there must exist a  $\theta \in (\theta^*, 1]$  such that  $\tilde{x}_S(\theta, (\theta^*, 1]) = c$ . Let  $\theta'_c$  and  $\theta''_c$  be such deviation types for the two cutoffs  $\theta'$  and  $\theta''$ , respectively. First, I claim that  $\theta'_c < \theta''_c$ . Suppose not, and  $\theta'_c > \theta''_c$ . Then,  $\tilde{x}_S(\theta''_c, (\theta', 1]) < \tilde{x}_S(\theta'_c, (\theta', 1]) = c$ . But by single-crossing property,  $\tilde{x}_S(\theta''_c, (\theta', 1]) > \tilde{x}_S(\theta''_c, (\theta'', 1]) = c$ . This is a contradiction. Therefore the claim holds. In order to prove the result for the sender, I will have to consider two possible scenarios.

**Scenario (a):** When  $\theta'_c < \theta''$ . That is,  $\theta' < \theta'_c < \theta'' < \theta''_c$ . The sender's utility

under the two PRE's is given by,

$$\begin{aligned}
PRE' : V_S(\theta') &= \int_0^{\theta'} U(\phi^S(x_S^*(t), x_R^*(t)), t, b) f(t) dt + \\
&\quad \int_{\theta'}^1 U(\phi^S(x_S^*(t, (\theta', 1]), x_R^*((\theta', 1])), t, b) f(t) dt \\
PRE'' : V_S(\theta'') &= \int_0^{\theta''} U(\phi^S(x_S^*(t), x_R^*(t)), t, b) f(t) dt + \\
&\quad \int_{\theta''}^1 U(\phi^S(x_S^*(t, (\theta'', 1]), x_R^*((\theta'', 1])), t, b) f(t) dt
\end{aligned}$$

Under  $PRE'$  the sender is able to compensate efficiently in the interval  $(0, \theta'_c]$ . The same can be achieved in the equilibrium  $PRE''$ . What is left to be checked are those states in which there is inefficiency because of the bounds imposed on actions of the sender. In  $PRE'$  this corresponds to the interval  $(\theta'_c, 1]$ . On the same interval, I compare the utility (ex-ante) achieved under  $PRE''$ . I will refer to this utility as the residual welfare that results from inefficiency,  $V_S^{RES}(\theta^*)$ .

$$V_S^{RES}(\theta') = \int_{\theta'_c}^{\theta''} U(\phi^S(c, x_R^*((\theta', 1])), t, b) f(t) dt + \int_{\theta''}^1 U(\phi^S(c, x_R^*((\theta', 1])), t, b) f(t) dt$$

$$\begin{aligned}
V_S^{RES}(\theta'') &= \\
&\int_{\theta'_c}^{\theta''} U(\phi^S(x_S^*(t), x_R^*(t)), t, b) f(t) dt + \int_{\theta''}^{\theta'_c} U(\phi^S(x_S^*(t, (\theta'', 1]), x_R^*((\theta'', 1])), t, b) f(t) dt \\
&\quad + \int_{\theta'_c}^1 U(\phi^S(c, x_R^*((\theta'', 1])), t, b) f(t) dt
\end{aligned}$$

$$\begin{aligned}
& > \int_{\theta'_c}^{\theta''} U(\phi^S(x_S^*(t), x_R^*(t)), t, b) f(t) dt + \int_{\theta''}^{\theta''_c} U(\phi^S(c, x_R^*((\theta'', 1])), t, b) f(t) dt \\
& \quad + \int_{\theta''_c}^1 U(\phi^S(c, x_R^*((\theta'', 1])), t, b) f(t) dt \\
& > \int_{\theta'_c}^{\theta''} U(\phi^S(x_S^*(t), x_R^*(t)), t, b) f(t) dt + \int_{\theta''}^1 U(\phi^S(c, x_R^*((\theta'', 1])), t, b) f(t) dt \\
& > \int_{\theta'_c}^{\theta''} U(\phi^S(x_S^*(t), x_R^*(t)), t, b) f(t) dt + \int_{\theta''}^1 U(\phi^S(c, x_R^*((\theta', 1])), t, b) f(t) dt \\
& > \int_{\theta'_c}^{\theta''} U(\phi^S(c, x_R^*((\theta', 1])), t, b) f(t) dt + \int_{\theta''}^1 U(\phi^S(c, x_R^*((\theta', 1])), t, b) f(t) dt \\
& = V_S^{RES}(\theta')
\end{aligned}$$

The first inequality follows from the fact that on the interval  $(\theta'', \theta''_c)$ , the sender is able to compensate efficiently. The third follows from  $\phi^S(c, x_R^*((\theta', 1])) < \phi^S(c, x_R^*((\theta'', 1]))$  because  $x_R^*((\theta', 1]) < x_R^*((\theta'', 1])$  and positive spillover at the bound for  $S$ ,  $U_1|_{\hat{x}_S(\cdot) > c} > 0$ . Final inequality results from introducing inefficiency and the concavity assumption  $U_{11} < 0$ .

**Scenario (b):** When  $\theta'_c > \theta''$ . That is,  $\theta' < \theta'' < \theta'_c < \theta''_c$ .

In this case, as earlier, I will compare those states in which there is inefficiency generated by information pooling. As before, I compare the residual welfare.

$$V_S^{RES}(\theta') = \int_{\theta'_c}^{\theta''_c} U(\phi^S(c, x_R^*((\theta', 1])), t, b) f(t) dt + \int_{\theta''_c}^1 U(\phi^S(c, x_R^*((\theta', 1])), t, b) f(t) dt$$



$$V_S^{RES}(\theta'') = \int_{\theta'_c}^{\theta''_c} U(\phi^S(x_S^*(t, (\theta'', 1]), x_R^*((\theta'', 1])), t, b) f(t) dt +$$

$$\int_{\theta''_c}^1 U(\phi^S(c, x_R^*((\theta'', 1])), t, b) f(t) dt$$

Pairwise comparison yields,

$$\int_{\theta'_c}^{\theta''_c} U(\phi^S(x_S^*(t, (\theta'', 1]), x_R^*((\theta'', 1])), t, b) f(t) dt > \int_{\theta'_c}^{\theta''_c} U(\phi^S(c, x_R^*((\theta', 1])), t, b) f(t) dt$$

and,

$$\int_{\theta'_c}^1 U(\phi^S(c, x_R^*((\theta'', 1])), t, b) f(t) dt > \int_{\theta''_c}^1 U(\phi^S(c, x_R^*((\theta', 1])), t, b) f(t) dt$$

The first inequality follows from the inefficiency of contributing  $c$  on the interval  $(\theta'_c, \theta''_c]$  when instead sender can best respond to  $x_R^*((\theta'', 1])$ . The second inequality results from the positive spillover property in the interval  $(\theta''_c, 1]$  and the fact that  $x_R^*((\theta'', 1]) > x_R^*((\theta', 1])$ . Therefore,  $V_S^{RES}(\theta'') > V_S^{RES}(\theta')$ . This completes the proof.

### B.2.5 Proof of Proposition 13

I will prove this by making pairwise comparison between two thresholds  $\bar{\theta}$  and  $\theta' (< \bar{\theta})$ . From Proposition 4, we know that  $\theta'_c < \bar{\theta}_c$ . As before, there are two scenarios to consider.

**Scenario (a):** When  $\theta'_c < \bar{\theta}$ . That is,  $\theta' < \theta'_c < \bar{\theta} < \bar{\theta}_c$ .

In this case, every type  $\theta \in [0, \theta'_c]$  is indifferent between the two threshold equilibria, since the optimal actions are within the bound in both cases. Therefore,  $U(\phi^S(x_S^*(\theta, (\theta', 1]), x_R^*((\theta', 1])), \theta, b) = U(\phi^S(x_S^*(\theta, (\bar{\theta}, 1]), x_R^*((\bar{\theta}, 1])), \theta, b)$  since  $\tilde{x}_S(\theta, (\theta', 1]), \tilde{x}_S(\theta, (\bar{\theta}, 1]) \leq c$ . However, every  $\theta \in (\theta'_c, 1]$  strictly prefers the  $\bar{\theta}$  threshold equilibrium. To see this, let us further divide the interval  $(\theta'_c, 1]$  to  $(\theta'_c, \bar{\theta}_c]$  and  $(\bar{\theta}_c, 1]$ . Now, every  $\theta \in (\theta'_c, \bar{\theta}_c]$  prefers the threshold  $\bar{\theta}$  since  $\tilde{x}_S(\theta, (\bar{\theta}, 1]) \leq c$ ,

whereas with threshold  $\theta'$ ,  $\tilde{x}_S(\theta, (\theta', 1]) > c$  implying that the constrained action is  $x_S^*(\theta, (\theta', 1]) = c$ . Therefore, for  $\theta \in (\theta'_c, \bar{\theta}_c]$ ,  $U(\phi^S(c, x_R^*((\theta', 1])), \theta, b) < U(\phi^S(x_S^*(\theta, (\bar{\theta}, 1]), x_R^*((\bar{\theta}, 1])), \theta, b)$ .

Lastly, for types  $\theta \in (\bar{\theta}_c, 1]$ , the unconstrained action under both the thresholds are above  $c$ . This means  $\tilde{x}_S(\theta, (\theta', 1]), \tilde{x}_S(\theta, (\bar{\theta}, 1]) > c$ . But,  $R$ 's action is higher under  $\bar{\theta}$  ( $x_R^*((\bar{\theta}, 1]) > x_R^*((\theta', 1])$ ) and due to the positive spillover property, it follows that  $U(\phi^S(c, x_R^*((\theta', 1])), \theta, b) < U(\phi^S(c, x_R^*((\bar{\theta}, 1])), \theta, b)$  for all  $\theta \in (\bar{\theta}_c, 1]$ .

**Scenario (b):** When  $\theta'_c < \bar{\theta}$ . That is,  $\theta' < \bar{\theta} < \theta'_c < \bar{\theta}_c$ .

A analogous set of arguments hold true for this case. In particular, every type  $\theta \in [0, \theta'_c]$  is indifferent between the thresholds  $\bar{\theta}$  and  $\theta'$ . Every type  $\theta \in (\theta'_c, \bar{\theta}_c]$  are strictly better off under threshold  $\bar{\theta}$  because  $\tilde{x}_S(\theta, (\bar{\theta}, 1]) \leq c$ , whereas with threshold  $\theta'$ ,  $\tilde{x}_S(\theta, (\theta', 1]) > c$ . Types  $\theta \in (\bar{\theta}_c, 1]$  are also strictly better off under threshold  $\bar{\theta}$  because of the positive spillover argument made earlier. This completes the proof.

## Appendix C

# An Informational Theory of Alliance Formation

### C.1 Full Information Aggregation

Before proceeding to prove Lemma 1, we provide some basic insights into the nature of maximization problem that each type of players face, and in general, lay out some important properties of the Beta-Binomial distribution that we employ in our paper. We start by reformulating the maximization problem faced by a truthful player, given in equation 3.3.1, as follows:

$$\max_{x_i} \int_0^1 \sum_{s_B \in \{0,1\}^{n-t}} u_i((x_i, x_{T \setminus \{i\}}, x_B(s_B)); \theta, b_i) \Pr(s_B | \theta) f(\theta | m_T) d\theta$$

The conditional density  $f(\theta | m_T)$  belongs to a standard beta-binomial distribution. Letting  $k = \sum_{i \in T} s_i$ , the number of signals  $s_i$  with  $i \in T$  that are equal to one, the posterior distribution of  $\theta$  with uniform prior on  $[0, 1]$ , given  $k$  successes in  $t$  trials, is a Beta distribution with parameters  $k + 1$  and  $t - k + 1$ . As a consequence,  $f(\theta | m_T) = \frac{(t+1)!}{k!(t-k)!} \theta^k (1 - \theta)^{t-k}$  and  $E[\theta | m_T] = [k + 1] / [t + 2]$ . Further, for any  $s_B$ , letting  $\ell(s_B) = \sum_{q \in B} s_q$ , it is the case that  $\Pr(s_B | \theta) = \theta^{\ell(s_B)} (1 - \theta)^{n-t-\ell(s_B)}$ .

In a similar way, the problem of every babbling player  $j \in B$  with a private signal  $s_j$ , stated in equation 3.3.2, can be expanded as the following:

$$\max_{x_j(s_j)} \int_0^1 \sum_{s_{B \setminus \{j\}} \in \{0,1\}^{n-t-1}} u_j((x_j(s_j), x_T, x_{B \setminus \{j\}}(s_{B \setminus \{j\}})); \theta, b_j) \Pr(s_{B \setminus \{j\}} | \theta) f(\theta | m_T, s_j) d\theta$$

Again, the posterior density  $f(\theta | m_T, s_j)$  belongs to the beta family, with  $k + s_j$  successes in  $t + 1$  signals, and is a Beta distribution with parameters  $k + s_j + 1$  and  $(t - k - s_j + 2)$ . Consequently,  $f(\theta | m_T, s_j) = \frac{(t+2)!}{(k+s_j)!(t+1-k-s_j)!} \theta^{k+s_j} (1-\theta)^{t+1-k-s_j}$  and  $E[\theta | m_T, s_j] = [k + s_j + 1]/[t + 3]$ . As before, for any  $s_{B \setminus \{j\}}$ ,  $\Pr(s_{B \setminus \{j\}} | \theta) = \theta^{\ell(s_{B \setminus \{j\}})} (1-\theta)^{n-t-\ell(s_{B \setminus \{j\}})}$ .

### C.1.1 Characterization of second-stage contributions with unrestricted domain

We begin the characterization by first solving the best responses of each of the three types of players from equations 3.3.1 and 3.3.2.

i) Truthful player's problem:

$$E_\theta[u_i(\mathbf{x}, \mathbf{m})] = - \int_0^1 \sum_{s_B \in \{0,1\}^{n-t}} \left( \frac{x_i + \eta \sum_{j \in T \setminus \{i\}} x_j + \eta \sum_{j \in B} x_j(s_j)}{1 + (n-1)\eta} - \theta - b_i \right)^2 \Pr(s_B | \theta) f(\theta | m_T) d\theta$$

where  $f(\theta | m_T) = \frac{(t+1)!}{k!(t-k)!} \theta^k (1-\theta)^{t-k}$ , iff  $0 \leq \theta \leq 1$ .

Differentiating the above with respect to  $x_i$ , we get the following FOC:

$$\int_0^1 \sum_{s_B \in \{0,1\}^{n-t}} \left( \frac{x_i + \eta \sum_{j \in T \setminus \{i\}} x_j + \eta \sum_{j \in B} x_j(s_j)}{1 + (n-1)\eta} - \theta - b_i \right) \Pr(s_B | \theta) f(\theta | m_T) d\theta = 0$$

Simplifying, we obtain:

$$x_i + \eta \left[ \sum_{j \in T \setminus \{i\}} x_j + \int_0^1 \sum_{s_B \in \{0,1\}^{n-t}} \sum_{j \in B} x_j(s_j) \Pr(s_B|\theta) f(\theta|m_T) d\theta \right] = \quad (C.1.1)$$

$$(b_i + E[\theta|m_T]) [1 + (n-1)\eta]$$

ii) *Babbling player's problem:*

With analogous procedures, the expected utility of a babbling player  $i$  with signal  $s_i$  is:

$$\begin{aligned} E_\theta[u_i(\mathbf{x}, \mathbf{m})] &= -E_{\theta, s_{-i}} \left[ \left( \frac{x_i(s_i) + \eta \sum_{j \neq i} x_j(s_j)}{1 + (n-1)\eta} - \theta - b_i \right)^2 \mid m_T, s_i \right] \\ &= -E_{\theta, s_{B \setminus \{i\}}} \left[ \left( \frac{x_i(s_i) + \eta \sum_{j \in T} x_j + \eta \sum_{j \in B \setminus \{i\}} x_j(s_j)}{1 + (n-1)\eta} - \theta - b_i \right)^2 \mid m_T, s_i \right] \\ &= -\int_0^1 \sum_{s_{B \setminus \{i\}} \in \{0,1\}^{n-t-1}} \left( \frac{x_i(s_i) + \eta \sum_{j \in T} x_j + \eta \sum_{j \in B \setminus \{i\}} x_j(s_j)}{1 + (n-1)\eta} - \theta - b_i \right)^2 \Pr(s_{B \setminus \{i\}}|\theta) \\ &\quad f(\theta|m_T, s_i) d\theta \end{aligned}$$

Again, the density  $f(\theta|m_T, s_i)$  belongs to the beta family such that  $f(\theta|m_T, s_i) = \frac{(t+2)!}{(k+s_i)!(t+1-k-s_i)!} \theta^{k+s_i} (1-\theta)^{t+1-k-s_i}$ , iff  $0 \leq \theta \leq 1$ . Differentiating the above equation, we derive the following

FOC:

$$\int_0^1 \sum_{s_{B \setminus \{i\}} \in \{0,1\}^{n-t-1}} \left( \frac{x_i(s_i) + \eta \sum_{j \in T} x_j + \eta \sum_{j \in B \setminus \{i\}} x_j(s_j)}{1 + (n-1)\eta} - \theta - b_i \right) \Pr(s_{B \setminus \{i\}}|\theta) f(\theta|m_T, s_i) d\theta = 0$$

Simplifying yields,

$$x_i(s_i) + \eta \left[ \sum_{j \in T} x_j + \int_0^1 \sum_{s_{B \setminus \{i\}} \in \{0,1\}^{n-t-1}} \sum_{j \in B \setminus \{i\}} x_j(s_j) \Pr(s_{B \setminus \{i\}} | \theta) f(\theta | m_T, s_i) d\theta \right] =$$

$$(b_i + E[\theta | m_T, s_i]) [1 + (n-1)\eta] \quad (\text{C.1.2})$$

We focus on linear equilibrium strategies of the form:  $x_i = A.(b_i + E[\theta | m_T]) + B$  for truthful players, and  $x_i(s_i) = A_{s_i}.(b_i + E[\theta | m_T, s_i]) + B_{s_i}$  for babbling players. Plugging the linear forms into expression (C.1.1), we get the following,

$$A(b_i + E[\theta | m_T]) + B + \eta \sum_{j \in T \setminus \{i\}} [A(b_j + E[\theta | m_T]) + B]$$

$$+ \eta \int_0^1 \sum_{s_B \in \{0,1\}^{n-t}} \sum_{j \in B} [A_{s_j}(b_j + E[\theta | m_T, s_j]) + B_{s_j}] \Pr(s_B | \theta) f(\theta | m_T) d\theta =$$

$$(b_i + E[\theta | m_T]) [1 + (n-1)\eta]$$

Using linearity of the strategies  $x_i(s_i)$ , above expression can be rewritten as:

$$[A(b_i + E[\theta | m_T]) + B] + \eta \sum_{j \in T \setminus \{i\}} [A(b_j + E[\theta | m_T]) + B]$$

$$+ \eta \int_0^1 \sum_{j \in B} \sum_{s_j \in \{0,1\}} [A_{s_j}(b_j + E[\theta | m_T, s_j]) + B_{s_j}] \Pr(s_j | \theta) f(\theta | m_T) d\theta =$$

$$(b_i + E[\theta | m_T]) [1 + (n-1)\eta]$$

Substituting in the functional forms of  $\Pr(s_j | \theta)$  and  $f(\theta | m_T, s_i)$ , we obtain:

$$\begin{aligned}
& (b_i + E[\theta|m_T]) [1 + (n-1)\eta] = \\
& [A(b_i + E[\theta|m_T]) + B] + \eta \sum_{j \in T \setminus \{i\}} [A(b_j + E[\theta|m_T]) + B] \\
& + \eta \int_0^1 \sum_{j \in B} [A_0(b_j + E[\theta|m_T, s_j = 0]) + B_0] (1-\theta)^{\frac{(t+1)!}{k!(t-k)!}} \theta^k (1-\theta)^{t-k} d\theta \\
& + \eta \int_0^1 \sum_{j \in B} [A_1(b_j + E[\theta|m_T, s_j = 1]) + B_1] \theta^{\frac{(t+1)!}{k!(t-k)!}} \theta^k (1-\theta)^{t-k} d\theta
\end{aligned}$$

which, because  $\int_0^1 (1-\theta)^{\frac{(t+1)!}{k!(t-k)!}} \theta^k (1-\theta)^{t-k} d\theta = 1 - E[\theta|m_T]$  and  $\int_0^1 \theta^{\frac{(t+1)!}{k!(t-k)!}} \theta^k (1-\theta)^{t-k} d\theta = E[\theta|m_T]$  is further simplified as:

$$\begin{aligned}
& (b_i + E[\theta|m_T]) [1 + (n-1)\eta] = A(b_i + E[\theta|m_T]) + B \\
& + \eta \sum_{j \in T \setminus \{i\}} [A(b_j + E[\theta|m_T]) + B] \\
& + \eta \sum_{j \in B} [A_0(b_j + E[\theta|m_T, s_j = 0]) + B_0] (1 - E[\theta|m_T]) \\
& + \eta \sum_{j \in B} [A_1(b_j + E[\theta|m_T, s_j = 1]) + B_1] E[\theta|m_T]
\end{aligned}$$

Substituting back the linear strategies  $x_i$  and  $x_j(s_j)$  gives the best response for the truthful players,

$$\begin{aligned}
(b_i + E[\theta|m_T]) [1 + (n-1)\eta] = x_i + \eta \sum_{j \in T \setminus \{i\}} x_j + \eta (1 - E[\theta|m_T]) \sum_{j \in B} x_j(0) + \\
\eta E[\theta|m_T] \sum_{j \in B} x_j(1)
\end{aligned}$$

Applying the same principles to equation (C.1.2), we obtain the expression:

$$\begin{aligned}
& (b_i + E[\theta|m_T, s_i]) [1 + (n-1)\eta] = \\
& A_{s_i}(b_i + E[\theta|m_T, s_i]) + B_{s_i} + \eta \sum_{j \in T} [A(b_j + E[\theta|m_T]) + B] \\
& + \eta \int_0^1 \sum_{s_{B \setminus \{i\}} \in \{0,1\}^{n-t-1}} \sum_{j \in B \setminus \{i\}} [A_{s_j}(b_j + E[\theta|m_T, s_j]) + B_{s_j}] \Pr(s_{B \setminus \{i\}}|\theta) f(\theta|m_T, s_i) d\theta
\end{aligned}$$

The manipulations on this equation are analogous in that we did previously. Hence, performing similar substitutions, we obtain the expression:

$$\begin{aligned}
(b_i + E[\theta|m_T, s_i]) [1 + (n-1)\eta] &= A_{s_i}(b_i + E[\theta|m_T, s_i]) + B_{s_i} \\
&+ \eta \sum_{j \in T} [A(b_j + E[\theta|m_T]) + B] \\
&+ \eta \sum_{j \in B \setminus \{i\}} [A_0(b_j + E[\theta|m_T, s_j = 0]) + B_0] (1 - E[\theta|m_T, s_i]) \\
&+ \eta \sum_{j \in B \setminus \{i\}} [A_1(b_j + E[\theta|m_T, s_j = 1]) + B_1] E[\theta|m_T, s_i]
\end{aligned}$$

which, again, gives us the following FOC for babbling players with private signal  $s_i$  ( $= 0$  or  $1$ )

$$\begin{aligned}
(b_i + E[\theta|m_T, s_i]) [1 + (n-1)\eta] &= x_i(s_i) + \eta \sum_{j \in T} x_j + \eta (1 - E[\theta|m_T, s_i]) \sum_{j \in B \setminus \{i\}} x_j(0) \\
&+ \eta E[\theta|m_T, s_i] \sum_{j \in B \setminus \{i\}} x_j(1)
\end{aligned}$$

Together, we can sum up the best responses for the three types of players as the following:



Truthful player  $i \in T$  -

$$\begin{aligned}
x_i = (b_i + E[\theta|m_T]) [1 + (n-1)\eta] - \eta \sum_{j \in T \setminus \{i\}} x_j - \eta (1 - E[\theta|m_T]) \sum_{j \in B} x_j(0) \\
- \eta E[\theta|m_T] \sum_{j \in B} x_j(1)
\end{aligned} \tag{C.1.3}$$

Babbling player with low signal  $i \in B$ ,  $s_i = 0$  -

$$\begin{aligned}
x_i(0) = (b_i + E[\theta|m_T, 0]) [1 + (n-1)\eta] - \eta \sum_{j \in T} x_j - \eta (1 - E[\theta|m_T, 0]) \sum_{j \in B \setminus \{i\}} x_j(0) \\
- \eta E[\theta|m_T, 0] \sum_{j \in B \setminus \{i\}} x_j(1)
\end{aligned} \tag{C.1.4}$$

Babbling player with high signal  $i \in B$ ,  $s_i = 1$  -

$$\begin{aligned}
x_i(1) = (b_i + E[\theta|m_T, 1]) [1 + (n-1)\eta] - \eta \sum_{j \in T} x_j - \eta (1 - E[\theta|m_T, 1]) \sum_{j \in B \setminus \{i\}} x_j(0) \\
- \eta E[\theta|m_T, 1] \sum_{j \in B \setminus \{i\}} x_j(1)
\end{aligned} \tag{C.1.5}$$

To verify if the equilibrium actions dictated by Lemma 1 is indeed right, we substitute them into the RHS of each of the above three equations C.1.3, C.1.4 and C.1.5.

Take equation C.1.3 :

$$\begin{aligned}
x_i &= (b_i + E[\theta|m_T]) [1 + (n-1)\eta] - \eta \sum_{j \in T \setminus \{i\}} x_j - \eta (1 - E[\theta|m_T]) \sum_{j \in B} x_j(0) - \\
&\eta E[\theta|m_T] \sum_{j \in B} x_j(1)
\end{aligned}$$

$$\begin{aligned}
x_i &= (b_i + E[\theta|m_T]) [1 + (n-1)\eta] - \eta \sum_{j \in T \setminus \{i\}} \frac{(1 + (n-1)\eta)}{1 - \eta} b_j + \frac{(t-1)\eta^2}{(1-\eta)} \sum_{g \in N} b_g \\
&\quad - \eta \cdot (t-1) \frac{(k+1)}{(t+2)} - \eta \sum_{j \in B} \frac{(1 + (n-1)\eta)}{1 - \eta} b_j + \frac{b \cdot \eta^2}{(1-\eta)} \sum_{g \in N} b_g \\
&\quad - \eta \cdot b \cdot \frac{(k+1)}{(t+2)} \cdot \frac{h(t)}{1 + h(t)} - \eta \cdot b \cdot E[\theta|m_T] \cdot \frac{1}{1 + h(t)}
\end{aligned}$$

Making the substitution that  $E[\theta|m_T] = \frac{(k+1)}{(t+2)}$ , we get,

$$\begin{aligned}
x_i &= [1 + (n-1)\eta] b_i + \frac{(k+1)}{(t+2)} + (n-1)\eta \frac{(k+1)}{(t+2)} - \eta \sum_{j \in N \setminus \{i\}} \frac{(1 + (n-1)\eta)}{1 - \eta} b_j \\
&\quad + \frac{(n-1)\eta^2}{(1-\eta)} \sum_{g \in N} b_g - \eta \cdot (n-1) \frac{(k+1)}{(t+2)} \\
x_i &= [1 + (n-1)\eta] b_i + \frac{(k+1)}{(t+2)} - \frac{\eta}{1-\eta} \sum_{j \in N \setminus \{i\}} b_j - \frac{(n-1)\eta^2}{1-\eta} \sum_{j \in N \setminus \{i\}} b_j \\
&\quad + \frac{(n-1)\eta^2}{(1-\eta)} b_i + \frac{(n-1)\eta^2}{(1-\eta)} \sum_{j \in N \setminus \{i\}} b_j \\
\implies x_i &= \frac{(1 + (n-2)\eta)}{1 - \eta} b_i - \frac{\eta}{1 - \eta} \sum_{j \in N \setminus \{i\}} b_j + \frac{(k+1)}{(t+2)}
\end{aligned}$$

The above equation can be rewritten as,

$$x_i = \frac{(1 + (n-1)\eta)}{1 - \eta} [b_i - \frac{\eta}{(1 + (n-1)\eta)} \sum_{j \in N} b_j] + \frac{(k+1)}{(t+2)}$$

Similar substitutions and simplification yields the equilibrium actions of the babbling types from their best response equations C.1.4 and C.1.5. This completes the proof.

### C.1.2 Proof of Theorem 1

*Necessity:* As argued in Section 3, a 0 – *type* player always reveals the low signal and the 1 – *type* player never misreports a high signal. The only cases of relevance then is one where 0 – *type* (1 – *type*) gets a high (low) signal.

Take the case of a 0 – *type* player. For  $i$  to reveal a high signal  $s_i = 1$ , it must be that, for any possible realization of the other  $(n - 1)$  players' signals, sending a truthful message  $m_i = s_i = 1$  must be optimal. This means that the equilibrium action of  $i$ ,  $x_i(1, 1, m_{-i}) \geq 0$  for any set of (truthful) messages from the other players,  $m_{-i}$ . Since the posterior on the state  $\theta$  is a beta-binomial distribution, what matters is the sufficient statistic  $k$ , the number of 1's in the set of messages  $(m_i, m_{-i})$ .

Therefore, for  $i$  to reveal  $s_i = 1$ , a set of  $n$  constraints (corresponding to  $k = 1$  to  $n$ ). However, the tightest constraint that would ensure this is when every other player reveals 0, meaning that  $\sum m_{-i} = 0$ . In this case, if  $m_i = 1$ , then  $k = \sum_{j \in N} m_j = 1$  and therefore the expected value of  $\theta$ ,  $E[\theta \mid m] = \frac{2}{n+2}$ . Once this constraint is satisfied, every other *IC* for player  $i$  must be satisfied. From the equation 3.3.3, it must be that,

$$\begin{aligned} \frac{(1 + (n - 1)\eta)}{1 - \eta} \cdot A_i + \frac{2}{(n + 2)} &\geq 0 \\ A_{i \in 0\text{-type}} &\geq -\frac{2}{(n + 2)} \cdot \frac{(1 - \eta)}{(1 + (n - 1)\eta)} \end{aligned} \tag{C.1.6}$$

A similar argument ensues for a player  $j \in 1 - \text{type}$ . For  $i$  to reveal a low signal truthfully, it must be that for any other order of  $(n - 1)$  truthful signals from the other players, player  $i$ 's optimal action upon sending the message  $m_j = s_j = 0$  must be within the upper bound of the action set. As before, we only need to concentrate on the tightest *IC* that satisfies this condition. In the case of  $j$ , this is the constraint when  $\sum m_{-j} = (n - 1)$ , that is, every other player reveals a high signal. In this

case, if  $m_j = 0$ , then  $k = \sum_N m = (n - 1)$  and therefore the expected value of  $\theta$ ,  $E[\theta \mid m] = \frac{n}{n+2}$ . Once this constraint is satisfied, every other *IC* for player  $j$  must be satisfied. From the equation 3.3.3, it must be that,

$$\frac{(1 + (n - 1)\eta)}{1 - \eta} \cdot A_j + \frac{n}{(n + 2)} \leq 1$$

$$A_{j \in 1\text{-type}} \leq (1 - \frac{n}{(n + 2)}) \cdot \frac{(1 - \eta)}{(1 + (n - 1)\eta)} = \frac{2}{(n + 2)} \cdot \frac{(1 - \eta)}{(1 + (n - 1)\eta)} \quad (\text{C.1.7})$$

Since  $A_{i \in 0\text{-type}} < 0$  and  $A_{j \in 1\text{-type}} > 0$  by definition, by combining equations C.1.6 and C.1.7, we conclude that there is full information aggregation if:

$$A_{i \in N} \leq \frac{2}{(n + 2)} \cdot \frac{(1 - \eta)}{(1 + (n - 1)\eta)} \quad (\text{C.1.8})$$

*Sufficiency:*

We prove this by contradiction. Suppose there is a  $n$ -player equilibrium and also that for some player  $i \in N$ , condition C.1.8 is violated. Without loss of generality, let the condition be violated for player  $n$ , with conflict of interest  $b_n$ . Then, given that each of remaining  $(n - 1)$  players are being truthful, it requires to be checked if  $n$  has an incentive to report her signal. Since  $b_n = \sup\{b_i : i \in N\}$ ,  $n$  is a  $1\text{-type}$  player. Further, as before,  $s_n = 0$  and  $n$  reports truthfully. Then, if each of the other signals are such that  $\sum m_{-n} = (n - 1)$ , then the equilibrium action of  $n$  is  $x_n = \min\{1, \frac{(1 + (n - 1)\eta)}{1 - \eta} \cdot A_j + \frac{n}{(n + 2)}\} = 1$ , since condition C.1.8 is violated by our construction. This implies there is under-provision from  $n$ 's point of view.

Now instead, if  $n$  misreports her signal and sends a message  $m_n = 1 - s_n = 1$ , then the actions of every other player is increased in equilibrium to the following:

$$x_{j \in N \setminus \{n\}}(s_j, m_{-n}, 1) = \frac{(1 + (n - 1)\eta)}{1 - \eta} \cdot A_j + \frac{(n + 1)}{(n + 2)}$$

The above is the equilibrium action of every player other than  $n$ , who received a

signal  $s_j$ , received truthful messages from every other player apart from  $n$ ,  $m_{-n}$ , and receive the message  $m_n = 1$  from  $n$ . Letting the above expression to be within the bounds, meaning  $0 \leq x_{j \in N \setminus \{n\}}(s_j, m_{-n}, 1) \leq 1$ , this implies  $x_n$  is also modified according to  $n$ 's best response equation, given in C.1.3. Specifically,

$$x_n(s_n, m_{-n}, 1) = (b_n + E[\theta|m_{-n}, s_n]) [1 + (n-1)\eta] - \eta \sum_{j \in N \setminus \{n\}} x_j(s_j, m_{-n}, 1)$$

Substituting and simplifying yields the following revised action,

$$x_n(s_n, m_{-n}, 1) = \frac{(1 + (n-1)\eta)}{1 - \eta} \cdot A_j + \frac{n(1 - \eta) + \eta}{(n+2)}$$

The optimal action is therefore  $x_n = \min\{1, \frac{(1+(n-1)\eta)}{1-\eta} \cdot A_j + \frac{n(1-\eta)+\eta}{(n+2)}\}$ . It is easy to conclude that irrespective of whether  $x_n(s_n, m_{-n}, 1) \leq 1$  or not,  $n$  is better off since the actions of other players have unequivocally risen. Thus,  $n$  benefits from deviating to  $m_n = 1$  when  $s_n = 0$ . But if this is true, then a  $n$ -player equilibrium ceases to exist, contradicting the starting assumption. This concludes the proof.

# Bibliography

- [1] J. H. Aldrich. A downsian spatial model with party activism. *The American Political Science Review*, pages 974–990, 1983.
- [2] J. H. Aldrich. A spatial model with party activists: implications for electoral dynamics. *Public Choice*, 41(1):63–100, 1983.
- [3] J. H. Aldrich. *Why Parties?: a second look*. University of Chicago Press, 2011.
- [4] R. Alonso. Shared control and strategic communication. *Memo, Northwestern University*, 2007.
- [5] R. Alonso, W. Dessein, and N. Matouschek. When does coordination require centralization? *The American economic review*, 98(1):145–179, 2008.
- [6] R. Alonso and N. Matouschek. Optimal delegation. *The Review of Economic Studies*, 75(1):259–293, 2008.
- [7] E. Aragones and T. R. Palfrey. Mixed equilibrium in a downsian model with a favored candidate. *Journal of Economic Theory*, 103(1):131–161, 2002.
- [8] S. Ashworth and E. Bueno de Mesquita. Elections with platform and valence competition. *Games and Economic Behavior*, 67(1):191–216, 2009.
- [9] R. Aumann. Nash equilibria are not self-enforcing. *Economic Decision Making: Games, Econometrics and Optimisation*, pages 201–206, 1990.

- [10] D. Austen-Smith. Interest groups, campaign contributions, and probabilistic voting. *Public choice*, 54(2):123–139, 1987.
- [11] D. Austen-Smith. Information transmission in debate. *American Journal of political science*, pages 124–152, 1990.
- [12] D. Austen-Smith. Interested experts and policy advice: Multiple referrals under open rule. *Games and Economic Behavior*, 5(1):3–43, 1993.
- [13] D. Austen-Smith. Campaign contributions and access. *American Political Science Review*, pages 566–581, 1995.
- [14] S. Baliga and S. Morris. Co-ordination, spillovers, and cheap talk. *Journal of Economic Theory*, 105(2):450–468, 2002.
- [15] D. P. Baron. Service-induced campaign contributions and the electoral equilibrium. *The Quarterly Journal of Economics*, 104(1):45–72, 1989.
- [16] D. P. Baron. Electoral competition with informed and uninformed voters. *American Political Science Review*, pages 33–47, 1994.
- [17] M. Battaglini. Multiple referrals and multidimensional cheap talk. *Econometrica*, pages 1379–1401, 2002.
- [18] D. Bernhardt, J. Duggan, and F. Squintani. Electoral competition with privately-informed candidates. *Games and Economic Behavior*, 58(1):1–29, 2007.
- [19] D. Bernhardt, J. Duggan, and F. Squintani. The case for responsible parties. *American Political Science Review*, 103(4):570, 2009.
- [20] B. D. Bernheim and M. D. Whinston. Menu auctions, resource allocation, and economic influence. *The quarterly journal of economics*, pages 1–31, 1986.

- [21] T. Besley and S. Coate. An economic model of representative democracy. *The Quarterly Journal of Economics*, 112(1):85–114, 1997.
- [22] R. L. Calvert. Robustness of the multidimensional voting model: Candidate motivations, uncertainty, and convergence. *American Journal of Political Science*, pages 69–95, 1985.
- [23] Y. Chen, N. Kartik, and J. Sobel. Selecting cheap-talk equilibria. *Econometrica*, 76(1):117–136, 2008.
- [24] S. Coate. Political competition with campaign contributions and informative advertising. *Journal of the European Economic Association*, 2(5):772–804, 2004.
- [25] N. S. Concept. Active engagement, modern defence. *HTTP: <http://www.nato.int/lisbon2010/strategic-concept-2010-eng.pdf> (accessed on 5 December 2013).* *Strategic Concept for the Defence and Security of The Members of the North Atlantic Treaty Organisation, Active Engagement, Modern Defence*, 2010.
- [26] V. P. Crawford and J. Sobel. Strategic information transmission. *Econometrica: Journal of the Econometric Society*, pages 1431–1451, 1982.
- [27] R. J. Dalton and M. P. Wattenberg. *Parties Without Partisans: Political Change in Advanced Industrial Democracies. Comparative Politics*. Oxford Scholarship Online, 2000.
- [28] J. Farrell and R. Gibbons. Cheap talk with two audiences. *The American Economic Review*, 79(5):1214–1223, 1989.
- [29] J. Farrell and M. Rabin. Cheap talk. *The Journal of Economic Perspectives*, pages 103–118, 1996.
- [30] T. Feddersen and F. Gul. Polarization and income inequality: A dynamic model of unequal democracy. Technical report, Working Paper, 2014.



- [31] T. Feddersen and A. Sandroni. The calculus of ethical voting. *International Journal of Game Theory*, 35(1):1–25, 2006.
- [32] T. Feddersen and A. Sandroni. A theory of participation in elections. *The American Economic Review*, 96(4):1271–1282, 2006.
- [33] A. Galeotti, C. Ghiglino, and F. Squintani. Strategic information transmission networks. *Journal of Economic Theory*, 148(5):1751–1769, 2013.
- [34] A. S. Gerber and D. P. Green. The effects of canvassing, telephone calls, and direct mail on voter turnout: A field experiment. *American Political Science Review*, pages 653–663, 2000.
- [35] T. W. Gilligan and K. Krehbiel. Asymmetric information and legislative rules with a heterogeneous committee. *American Journal of Political Science*, pages 459–490, 1989.
- [36] G. M. Grossman and E. Helpman. Electoral competition and special interest politics. *The Review of Economic Studies*, 63(2):265–286, 1996.
- [37] G. M. Grossman and E. Helpman. Competing for endorsements. *American Economic Review*, pages 501–524, 1999.
- [38] F. Gul and W. Pesendorfer. Partisan politics and election failure with ignorant voters. *Journal of Economic Theory*, 144(1):146–174, 2009.
- [39] J. Hagenbach and F. Koessler. Strategic communication networks. *The Review of Economic Studies*, 77(3):1072–1099, 2010.
- [40] H. Herrera, D. K. Levine, and C. Martinelli. Policy platforms, campaign spending and voter participation. *Journal of Public Economics*, 92(3):501–513, 2008.
- [41] H. Herrera and C. Martinelli. Group formation and voter participation. *Theoretical Economics*, 1(4):461–487, 2006.

- [42] H. Herrera, M. Morelli, and S. Nunnari. Turnout across democracies. *American Journal of Political Science*, 2015.
- [43] H. Herrera, M. Morelli, and T. Palfrey. Turnout and power sharing. *The Economic Journal*, 124(574):F131–F162, 2014.
- [44] T. M. Holbrook and S. D. McClurg. The mobilization of core supporters: Campaigns, turnout, and electoral composition in united states presidential elections. *American Journal of Political Science*, 49(4):689–703, 2005.
- [45] N. Kartik. Strategic communication with lying costs. *The Review of Economic Studies*, 76(4):1359–1395, 2009.
- [46] N. Kartik, M. Ottaviani, and F. Squintani. Credulity, lies, and costly talk. *Journal of Economic theory*, 134(1):93–116, 2007.
- [47] V. Krishna and J. Morgan. A model of expertise. *Available at SSRN 150589*, 1999.
- [48] Z. Li, H. Rantakari, and H. Yang. Competitive cheap talk. *Games and Economic Behavior*, 96:65–89, 2016.
- [49] A. Madestam, D. Shoag, S. Veuger, and D. Yanagizawa-Drott. Do political protests matter? evidence from the tea party movement\*. *The Quarterly Journal of Economics*, 128(4):1633–1685, 2013.
- [50] N. McCarthy, K. T. Poole, and H. Rosenthal. *Polarized America*. Cambridge, MA: MIT Press.[End Page 791], 2006.
- [51] N. D. Melumad and T. Shibano. Communication in settings with no transfers. *The RAND Journal of Economics*, pages 173–198, 1991.
- [52] J. Morgan and P. C. Stocken. An analysis of stock recommendations. *RAND Journal of economics*, pages 183–203, 2003.

- [53] J. Morgan and P. C. Stocken. Information aggregation in polls. *The American Economic Review*, 98(3):864–896, 2008.
- [54] S. Morris. Political correctness. *Journal of political Economy*, 109(2):231–265, 2001.
- [55] R. B. Morton. Groups in rational turnout models. *American Journal of Political Science*, pages 758–776, 1991.
- [56] G. Moscarini. Competence implies credibility. *The American Economic Review*, 97(1):37–63, 2007.
- [57] P. Norris. *Democratic phoenix: Reinventing political activism*. Cambridge University Press, 2002.
- [58] P. Norris. Political activism: New challenges, new opportunities. *The Oxford handbook of comparative politics*, pages 628–652, 2007.
- [59] M. J. Osborne and A. Slivinski. A model of political competition with citizen-candidates. *The Quarterly Journal of Economics*, 111(1):65–96, 1996.
- [60] M. Ottaviani and F. Squintani. Naive audience and communication bias. *International Journal of Game Theory*, 35(1):129–150, 2006.
- [61] T. R. Palfrey and H. Rosenthal. A strategic calculus of voting. *Public Choice*, 41(1):7–53, 1983.
- [62] T. R. Palfrey and H. Rosenthal. Voter participation and strategic uncertainty. *American Political Science Review*, 79(01):62–78, 1985.
- [63] M. rAndon HersHey and P. A. Beck. Party politics in america. 2015.
- [64] H. Rantakari. Governing adaptation. *The Review of Economic Studies*, 75(4):1257–1285, 2008.

- [65] W. H. Riker and P. C. Ordeshook. A theory of the calculus of voting. *American political science review*, 62(01):25–42, 1968.
- [66] R. Shachar and B. Nalebuff. Follow the leader: Theory and evidence on political participation. *American Economic Review*, pages 525–547, 1999.
- [67] J. Sobel. A theory of credibility. *The Review of Economic Studies*, 52(4):557–573, 1985.
- [68] J. C. Stein. Cheap talk and the fed: A theory of imprecise policy announcements. *The American Economic Review*, pages 32–42, 1989.
- [69] S. M. Walt. Alliance formation and the balance of world power. *International security*, 9(4):3–43, 1985.
- [70] S. M. Walt. *The origins of alliance*. Cornell University Press, 1990.
- [71] K. N. Waltz. *Theory of international politics*. Waveland Press, 2010.
- [72] D. Wittman. Candidate motivation: A synthesis of alternative theories. *The American political science review*, pages 142–157, 1983.